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Abstract

We study the term structure of interest rates in an endowment economy with noisy information and CRRA preferences. Exogenous prices and consumption consist of both temporary and permanent components, but the household observes only their aggregate values. We show that on average the term spread in this environment is positive and on a scale close to what we observe in the data, a fact that many existing macroeconomic models struggle to reproduce without very large coefficients of relative risk aversion. In our partial-information framework, uncertainty about the decomposition of the endowment and prices into their temporary and permanent components combined with a negative correlation in consumption growth explain why the slope of the yield curve is positive on average. We estimate our model using Bayesian methods and US data from 1961–2007 and find that the average interest rate spread is 0.85%, compared with 0.98% in the data. Further, we estimate a coefficient of relative risk aversion of only 4.86. Noisy information accounts for 44% of the scale of the term premium, with the remainder principally explained by real activity and nominal factors playing only a small role.

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1 Introduction

Interest rates play an important role in macroeconomic models, representing an individual's willingness to transfer wealth into the future, attitude towards risk, and behaviour in response to uncertainty. For this reason, the ability of macroeconomic models to match some basic properties of interest rates—and asset prices in general—is an important consideration in model evaluation. Despite this attention, standard macroeconomic models produce long-term bond premia that are far too small compared with what we observe in the data. The models that are capable of replicating this feature of the data usually do so by using non-standard preferences—typically the recursive preferences proposed by Epstein and Zin (1989) or external habit formation as in Wachter (2006)—and often rely on a negative correlation between consumption and inflation and very large coefficients of relative risk aversion. Outside of macro-finance applications, however, these preferences are used considerably less than the standard Constant Relative Risk Aversion (CRRA) utility. While much has been learned from the existing approaches, a model with standard CRRA preferences capable of producing a sizable term premium remains an important topic given that these preferences feature in the vast majority of macroeconomic models and the vital importance of the term structure of interest rates for monetary authorities.

In this paper, we show that incorporating noisy information into a model with CRRA preferences can indeed reproduce a term spread on a scale close to what we observe in the data. In each period households receive an endowment consisting of both transitory and permanent components. The household, however, observes only the total endowment, not its individual components. This gives rise to a signal extraction problem as they forecast future economic activity to price bonds. Our model environment reflects the reality that individuals are inundated with imprecise and often contradictory information about current and future economic conditions. The exogenous process for the endowment implies that consumption growth is negatively correlated, which leads to a positive term premium as in Campbell (1986). Backus et al. (1989) consider a monetary extension of the Mehra and

Prescott (1985) model, showing that exchange models of this type cannot account for the scale of bond risk premia given the variability of consumption in the data. In our model, however, the signal extraction problem results in additional uncertainty which amplifies the household's forecast variance, leading to a larger slope of the yield curve compared with a full-information version of the model. The exogenous price level follows a process similar to the endowment and affects the yield curve in a similar manner. The term premium depends on the relative variances of the transitory and permanent components and the degree of risk aversion. Short-run economic fluctuations affect the slope of the yield curve and explain variations of the term structure over time.

We estimate the model using Bayesian methods and US bond yields from 1961–2007, which allows us to measure the contribution of the underlying causes behind the size and variability of the term premium.[1] We estimate an average spread between 10-year and 3-month yields of 0.85%, compared with 0.98% in the data. Not only are we able to match the scale of the term spread, our model also estimates a coefficient of relative risk aversion of only 4.86, considerably smaller than what is found in many term structure models. By comparison, a full-information model with perfect information produces an interest rate spread that is only about half the size, highlighting the important role for partial information in explaining the average size of the term premium. We then show that in terms of economic variables, real factors are the dominant force behind the size of the term premium, with nominal factors having only a small role. This is not because there is more variability in short-run real activity relative to prices, but rather because of the moderate value of risk aversion which amplifies the volatility of real activity in the term premium.

To generate a sizeable term premium requires a departure from standard macroeconomic frameworks. One branch of literature offers imperfect information as an explanation for an upward-sloping term premium, which is the approach we follow in this paper. Our paper is closely related to recent work by Tanaka (2024), who studies the countercyclicality of term

[1] We extend the sample until 2019 in Section 4.3. The shorter sample of our benchmark model avoids the zero lower bound period.

premia using a small-scale New Keynesian model with noisy information about productivity. Our partial equilibrium framework is distinct because we remain agnostic about the economic forces driving prices and consumption and consider a simpler noisy information mechanism. In fact, our model includes only a single structural equation—the Euler equation, the building block of the consumption-based asset pricing model. While Tanaka (2024) uses recursive preferences proposed by Epstein and Zin (1989), which requires a very large value for the coefficient of relative risk aversion to fit empirical moments, we are able to produce a sizable term spread using a simple model with standard CRRA preferences without relying on a large coefficient of risk aversion. Empirical estimates of the coefficient of relative risk aversion are found to be around 1, as shown by Chetty (2006). Another key advantage of our approach is that the estimated term spread remains substantial even when the correlation between consumption growth and inflation is positive, as is the case with demand shocks. The model's simplicity also enables us to derive analytically the role played by noisy information for the size of the term spread.

Other papers that focus on imperfect information include Kozicki and Tinsley (2005), Dewachter and Lyrio (2008), and Dewachter et al. (2011). Using a model with asymmetric information and learning dynamics, Kozicki and Tinsley (2005) show that departures from rational expectations can explain the rejection of the expectations hypothesis. Dewachter and Lyrio (2008) introduce departures from rational expectations into a New Keynesian model with habit formation and show that the model fits the term structure of interest rates and inflation expectations well. However, Dewachter et al. (2011) show that, while asymmetric information and learning may play an important role, on their own they do not appear sufficient to explain time variation in the term premium. In their learning dynamics model, the term premium is driven entirely by differences between perceived and actual inflation and interest rates. By contrast, in our Noisy Information model, expectations of all variables are affected by imperfect information.

A second branch of literature departs from the benchmark macroeconomic utility functions. Campbell and Cochrane (1999), Wachter (2006), and Christoffel et al. (2011) use models where preferences feature habit formation, so that individuals dislike large changes from previous levels of consumption. This approach can generate a positive term premium, but in many cases this result hinges on a negative covariance between inflation and consumption growth, and typically comes at the price of deteriorating model fit along other dimensions such as excess volatility of output, labor, and the real wage, as shown by Rudebusch and Swanson (2008).

Because of this limitation, other research, including Piazzesi and Schneider (2007), Rudebusch and Swanson (2012), Andreasen (2012), van Binsbergen et al. (2012), Kozak (2022), and Kliem and Meyer-Gohde (2022) uses the recursive preferences proposed by Epstein and Zin (1989). As with most of the models using habit formation, the term premium in these models hinges on a negative covariance between inflation and consumption. A recession caused by supply-side factors will typically increase inflation and decrease consumption. Higher-than-expected inflation erodes the return on long-term bonds, which makes them less valuable in periods of low consumption, exactly when households would like to use their savings to increase consumption. As a result, a positive term premium reflects a hedge against inflation risk. This same mechanism is at play in the model of rare disasters presented by Gabaix (2012). Of course, if the covariance between inflation and consumption is instead positive, as expected following demand shocks, the same intuition implies the term premium should be negative, so this mechanism seems unlikely to account for the fact that estimates of the term premium are nearly always positive.

Models featuring Epstein-Zin preferences also require a very large value for the coefficient of relative risk aversion, many times the typical estimates found in the data. This is not the case for the Noisy Information model, which can explain a substantial fraction of the term premium for only moderate values of risk aversion. This is true whether consumption and inflation are positively or negatively correlated, a desirable feature of the model given that the relationship between inflation and real activity characterized by the Phillips curve appears to have changed in the recent past, as demonstrated by Del Negro et al. (2020) and many other papers.

Finally, other models generate a non-trivial term premium by introducing financial market frictions. For example, Chen et al. (2012), Gertler and Karadi (2013), and Carlstrom et al. (2017) rely on market segmentation and preferred habitat environments to produce movements in the term premium. These models allow an important role for the supply of bonds to impact the term spread, consistent with recent empirical evidence for Germany by Billio et al. (2025). In these cases, however, the term premium is driven by the degree of market segmentation so that the causes behind the term premium lack a theoretical underpinning and hence remain unknown. Other analyses at the intersection of macroeconomics and finance of the term structure are summarized in Gürkaynak and Wright (2012).

The remainder of this paper proceeds as follows. In section 2 we present the noisy information model and derive the term premium for real and nominal bond yields, showing it is positive and increasing in bond maturity. Section 3 describes the data, the Bayesian estimation method, and reports parameter estimates. Section 4 compares bond yields predicted in the model from those in the data, presents the main results, and explores the sensitivity of our findings to alternative specifications. In Section 5, we report on the estimated degree of noisy information and show how it impacts the household's expectations relative to those of the econometrician. Section 6 concludes. Details of the derivations and estimation algorithm are included in the Appendix.

2 An endowment economy with noisy information

In this section, we present an endowment economy model with noisy information. Section 2.1 considers a simple univariate model for consumption and derives the term premium for real yields, which builds intuition for how noisy information affects the yield curve. Section 2.2 presents the benchmark multivariate model for consumption and prices, which we will estimate in later sections, and derives the yield spread for nominal bonds.

2.1 Yields on real bonds

We consider an endowment economy where a representative household receives C_t consumption goods in period t. The endowment consists of both permanent and temporary components, but the household observes only the total endowment. Because households do not observe the individual components, they first solve a signal extraction problem using the Kalman filter to forecast future consumption, which they use to price m-period bonds.

The log endowment, c_t, is the sum of a permanent component, $\tau_{c,t}$, which follows a random walk, and a white noise temporary component, $\eta_{c,t}$:

$$\begin{aligned} c_t &= \tau_{c,t} + \eta_{c,t}, \\ \tau_{c,t} &= \delta_c + \tau_{c,t-1} + \epsilon_{c,t}. \end{aligned} \tag{1}$$

The error terms, η_c and ϵ_c, are independent and normally distributed:

$$\begin{bmatrix} \eta_{c,t} \\ \epsilon_{c,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{\eta_c} & 0 \\ 0 & \sigma^2_{\epsilon_c} \end{bmatrix} \right). \tag{2}$$

Note that (1) implies that consumption growth is negatively serially correlated[2] with the strength of this correlation determined by the variance of the temporary component:

$$cov(\Delta c_{t+1}, \Delta c_t) = -\sigma^2_{\eta_c}.$$

All parameters are observed by the household, as is the aggregate value of consumption, but the temporary and permanent components are not individually observable. Here, aggregate consumption will act as a noisy signal of the underlying permanent component. This

[2]In Section 4.3 we relax this assumption by allowing for AR(2) dynamics so that consumption growth may be positively or negatively serially correlated.

complicates the household's forecasting problem, as we illustrate below.

The representative household has standard CRRA preferences, discounts future utility at rate ρ, and maximizes expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} e^{-\rho t} \frac{c_t^{1-\gamma} - 1}{1-\gamma},$$

where the parameter $\gamma \geq 0$ denotes the degree of risk aversion. The household prices m-period real bonds, $q_{t,m}^r$, using the Euler equation:

$$q_{t,m}^r = e^{-\rho m} C_t^{\gamma} E_t[C_{t+m}^{-\gamma}]. \tag{3}$$

When the temporary and permanent components are observable, the expressions above as well as the properties of the log-normal distribution can be used to get closed-form solutions for bond prices of arbitrary maturity. In our model, however, the household observes only total consumption, not its individual components. In this environment, which is similar to the model presented by Lorenzoni (2009), the household forecasts future consumption using the Kalman filter. Uncertainty over the decomposition of consumption into its temporary and permanent components gives rise to a yield curve that slopes upwards on average.

Despite the ubiquity of full-information rational expectations, models such as ours, where agents continue to behave rationally but have only partial information, have a long history in economics. Lucas (1972) presents a model where agents cannot distinguish between shifts of real and nominal demand, so that changes to the money supply can have real effects. This framework is extended by Woodford (2003) to include monopolistically competitive pricing and imperfect precision of others' subjective perceptions of the economy. Our environment is more similar to Kydland and Prescott (1982) and Tanaka (2024), who assume that technology consists of the sum of temporary and persistent components, but that agents observe only a noisy indicator of the total level of technology. Coibion and Gorodnichenko (2012, 2015), Beckmann and Reitz (2020), and Shintani and Ueda (2023) derive predictions about the forecast errors of agents to provide empirical support for noisy information models.

As shown in equation (3), the bond price is determined by current and forecasts of future consumption, which evolve as in (1). Because the individual components, $\tau_{c,t}$ and $\eta_{c,t}$, are unobserved, the household uses the Kalman filter to forecast future consumption. An implication of the Kalman filtering exercise is that expectations of the state variables evolve slowly in response to new information. When the Kalman filter has converged, expectations of the current permanent and temporary components based on period t information are:

$$\tau_{c,t|t} = \frac{1/\sigma^2_{\tau_c}}{1/\sigma^2_{\tau_c} + 1/\sigma^2_{\eta_c}}(\delta_c + \tau_{c,t-1|t-1}) + \frac{1/\sigma^2_{\eta_c}}{1/\sigma^2_{\tau_c} + 1/\sigma^2_{\eta_c}}c_t, \tag{4}$$

$$\eta_{c,t|t} = \frac{1/\sigma^2_{\tau_c}}{1/\sigma^2_{\tau_c} + 1/\sigma^2_{\eta_c}}(c_t - \delta_c - \tau_{c,t-1|t-1}), \tag{5}$$

where $\sigma^2_{\tau_c} \equiv Var_{t-1}(\tau_{c,t})$ is the solution to the Riccati equation: $\sigma^2_{\tau_c} = (1/\sigma^2_{\eta_c} + 1/\sigma^2_{\tau_c})^{-1} + \sigma^2_{\epsilon_c}$. Equation (4) shows that the forecast of the permanent component is a weighted average of prior and new information. As $\sigma^2_{\eta_c} \to 0$, c_t becomes a perfect signal of the trend component, so that the second term dominates and no weight is placed on old information. As $\sigma^2_{\eta_c} \to \infty$, c_t becomes instead a very noisy signal of $\tau_{c,t}$ so that the household places no weight on new information. And as in the standard Noisy Information model, expectations have a backward-looking component because households are slow to respond to new information.

Define $yld^r_{t,m} \equiv -m^{-1}\log q^r_{t,m}$ as the yield on an m-period real bond. From the properties of log-normal variables, we have:

$$yld^r_{t,m} = \rho + \gamma\delta_c - \frac{1}{m}\gamma\eta_{c,t|t} - \frac{1}{2}\frac{1}{m}\gamma^2(\sigma^2_{\tau_c} + (m-1)\sigma^2_{\epsilon_c} + \sigma^2_{\eta_c}), \tag{6}$$

which shows that real bond yields depend negatively on the household's expectation of the temporary component of consumption.

Using equation (6) to compare yields on an m- and one-period bond, we can show that the spread between short- and long-run bonds will be time-varying and on average positive:[3]

$$yld^r_{t,m} - yld^r_{t,1} = \underbrace{(1-1/m)\gamma}_{>0}\,\eta_{c,t|t} + \underbrace{0.5\gamma^2(1-1/m)(\sigma^2_{\eta_c} + (1/\sigma^2_{\eta_c} + 1/\sigma^2_{\tau_c})^{-1})}_{>0}. \tag{7}$$

The first term is time-varying and depends on the household's expectation of the current state of the economy. When $\eta_{c,t|t} > 0$, the household believes the economy is above trend and wants to transfer some of their endowment to the future, which lowers interest rates. Interest rates will then be expected to be higher in the future, when the endowment is expected to return to trend. This explains why the yield curve slopes upwards in this state. Likewise, when the economy is below trend, expected future interest rates will be less than the current interest rate and the real yield curve may be downward sloping or inverted. Short-run fluctuations of consumption can thus explain why the slope of the yield curve varies over time and can be positive or negative. Because $\eta_{c,t|t}$ is mean zero, however, this term alone cannot explain why the yield curve slopes upwards *on average*. The second term of equation (7) represents the term premium of the real yield curve.[4] It is time-invariant, positive, and increasing in maturity m, so that the yield curve will on average be upward sloping. The term premium is a positive function of the degree of risk aversion (γ), the variance of the temporary component ($\sigma^2_{\eta_c}$), and that of the filtered estimator of trend consumption ($\sigma^2_{\tau_c}$).

By comparison, under full-information, the difference between yields on an m- and one-period bond is:

$$(1-1/m)\gamma\eta_{c,t} + 0.5\gamma^2(1-1/m)\sigma^2_{\eta_c}. \tag{8}$$

The second term is again positive and increasing in maturity, indicating an upward sloping yield curve. The intuition behind this result is explained by Campbell (1986) and follows from the negative serial correlation in consumption growth. A positive endowment shock

[3]See appendix A for the derivation of real bond yields.
[4]See calculation in appendix B.

raises bond prices, which leads to a capital gain for long-run bond holders. But this capital gain occurs in a period when consumption is high, so that marginal utility is low. And, because consumption growth is negatively serially correlated, marginal utility is expected to be higher in the future, when consumption will be lower. Long-run bonds are thus undesirable to hold, which leads to a positive term premium. But this term alone cannot account for the scale of the term premium, as pointed out by Backus et al. (1989).

Comparing (7) and (8) we see that noisy information adds two features to the model. First, because the current transitory component of consumption is unobservable, $\eta_{c,t|t}$ will be slow to adjust to new information. Second, the term premium of the yield curve, which is the final term in equations (7) and (8), will be larger under partial information:

$$\underbrace{0.5\gamma^2(1-1/m)(\sigma^2_{\eta_c} + (1/\sigma^2_{\eta_c} + 1/\sigma^2_{\tau_c})^{-1})}_{\text{Partial info.}} > \underbrace{0.5\gamma^2(1-1/m)\sigma^2_{\eta_c}}_{\text{Full info.}}$$

which follows from:

$$0.5\gamma^2(1-1/m)(1/\sigma^2_{\eta_c} + 1/\sigma^2_{\tau_c})^{-1} > 0. \tag{9}$$

The term premium is larger under partial information because there is greater uncertainty about future economic variables when the current state variables are not observable. This amplifies the household's forecast variances, leading to a larger term premium for any given level of risk aversion, γ. Equation (9) shows that the variances of both the short- and long-run components matter under partial information, either directly or indirectly through $\sigma^2_{\tau_c}$. That is different from the full-information case, where the term premium of the yield curve is explained entirely by the variance of the short-run component.

2.2 Yields on nominal bonds

We now turn to the nominal yield curve. To price nominal bonds we need to model the joint behavior of consumption and prices. We now assume that both consumption and the aggregate price level, P_t, consist of permanent and temporary components. As in the previous subsection, the household observes only the total endowment and the overall price level, not their individual components.

The log price level, p_t, is the sum of a permanent component $\tau_{p,t}$, which follows a random walk, and a white noise temporary component, $\eta_{p,t}$:

$$\begin{aligned} p_t &= \tau_{p,t} + \eta_{p,t}, \\ \tau_{p,t} &= \delta_p + \tau_{p,t-1} + \epsilon_{p,t}. \end{aligned} \tag{10}$$

Consumption continues to evolve as in (1) and the errors, ϵ and η, are normally distributed:

$$\begin{pmatrix} \epsilon_{p,t} \\ \epsilon_{c,t} \\ \eta_{p,t} \\ \eta_{c,t} \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_\epsilon & 0 \\ 0 & \Sigma_\eta \end{bmatrix} \right), \tag{11}$$

where:

$$\Sigma_\epsilon = \begin{bmatrix} \sigma^2_{\epsilon_p} & \sigma_{\epsilon_p\epsilon_c} \\ \sigma_{\epsilon_p\epsilon_c} & \sigma^2_{\epsilon_c} \end{bmatrix} \text{ and } \Sigma_\eta = \begin{bmatrix} \sigma^2_{\eta_p} & \sigma_{\eta_p\eta_c} \\ \sigma_{\eta_p\eta_c} & \sigma^2_{\eta_c} \end{bmatrix}.$$

Consumption and prices are thus determined by a multivariate unobserved components model, a popular empirical model used to decompose macroeconomic variables into trend and cyclical components. Models of this type were first presented by Watson and Engle (1983) while Kuttner (1994) provides an early application to estimate potential output. Mitra and Sinclair (2012) show that estimates of latent state variables can be sensitive to assumptions about the correlation of their error terms, an extension of the same result for the univariate unobserved components model first shown by Morley et al. (2003). We allow for the two temporary components to be correlated, and likewise for the errors to the permanent components, but there is no correlation between the temporary and permanent components, as indicated by the block diagonal covariance matrix.

Using the structure of prices and consumption, the household prices m-period nominal bonds, $q^n_{t,m}$, using the Euler equation:

$$q^n_{t,m} = e^{-\rho m} C_t^{\gamma} P_t E_t[C_{t+m}^{-\gamma} P_{t+m}^{-1}]. \tag{12}$$

Prices and consumption evolve jointly according to equations (1), (10) and (11), which we can express compactly in matrix notation:

$$x_t = \Lambda \alpha_t \tag{13}$$

$$\alpha_t = \mu + T\alpha_{t-1} + u_t, \quad u_t \sim N(0, \Sigma_u), \tag{14}$$

where $x_t = [p_t, c_t]'$, $\alpha_t = [\tau_{p,t}, \tau_{c,t}, \eta_{p,t}, \eta_{c,t}]'$, and:.

$$\Lambda = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} I_2 & 0_{2\times 2} \\ 0_{2\times 2} & 0_{2\times 2} \end{bmatrix}.$$

From the Euler equation (12) and using $T^m = T$, the yield on a nominal bond maturing in m periods is:

$$yld^n_{t,m} = \rho + \delta_p + \gamma\delta_c - \frac{1}{m}\eta_{p,t|t} - \frac{1}{m}\gamma\eta_{c,t|t} - \frac{1}{2}\frac{1}{m}[1 \;\; \gamma]\Lambda M_{t+m|t}\Lambda'[1 \;\; \gamma]', \tag{15}$$

where $M_{t+m|t} = E[(\alpha_{t+m} - \hat{\alpha}_{t+m|t})(\alpha_{t+m} - \hat{\alpha}_{t+m|t})']$ is the mean squared prediction error of the state variables. We can write the difference between yields on long- and short-run nominal bonds as:

$$yld^n_{t,m} - yld^n_{t,1} = (1 - 1/m)(\eta_{p,t|t} + \gamma\eta_{c,t|t}) + 0.5[1 \;\; \gamma]\Lambda(M_{t+1|t} - 1/mM_{t+m|t})\Lambda'[1 \;\; \gamma]'. \quad (16)$$

The first term says that the yield curve will be upward sloping when either consumption or prices are above trend. This is analogous to the time-varying component in the yield spread for real bonds. Here, when η_p is positive, the temporary component of the price level is expected to be lower in the future when it returns to trend. The household thus anticipates deflationary pressure, which lowers current interest rates relative to expected future interest rates, steepening the yield curve. The second term is the term premium and will be positive because both consumption growth and growth in the price level are negatively serially correlated. And as before, partial information leads to additional uncertainty and hence larger forecast variances, which amplifies this term compared with the full-information case.

3 Estimation details

3.1 Data

We estimate the model using quarterly US data over the period June 1961 to December 2007. We use the BEA's series of personal consumption expenditures, deflated by population and the CPI, as our measure of consumption, and the CPI is our measure of prices. Both variables enter the state space model in logs. We use constant-maturity zero-coupon Treasury yield curve data from Liu and Wu (2021) at maturities $m = 3, 6, 12, 18, 24, 36, 48, 60, 84,$ and 120 months.

3.2 State space representation

The presence of partial information complicates estimation because neither the model's household nor the econometrician observes the temporary and permanent components of consumption and prices. We follow the approach proposed by Blanchard et al. (2013), which involves treating the household's filtered expectations of these components $(\tau_{p,t|t}, \tau_{c,t|t}, \eta_{p,t|t}, \eta_{c,t|t})$ as additional state variables. The state vector is then:

$$\underset{\sim}{\alpha}_t = [\tau_{p,t},\ \tau_{c,t},\ \eta_{p,t},\ \eta_{c,t},\ \tau_{p,t|t},\ \tau_{c,t|t},\ \eta_{p,t|t},\ \eta_{c,t|t}]', \tag{17}$$

which evolves as:

$$\underset{\sim}{\alpha}_t = \underset{\sim}{\mu} + \underset{\sim}{T}\,\underset{\sim}{\alpha}_{t-1} + \underset{\sim}{R}u_t. \tag{18}$$

Notice we use a $\underset{\sim}{}$ to distinguish the econometrician's state variables and parameter matrices from those of the household. The measurement equations map the observable variables—consumption, prices, and bond yields—to the state variables:

$$\underset{\sim}{x}_t = \underset{\sim}{d} + \underset{\sim}{\Lambda}\,\underset{\sim}{\alpha}_t + \underset{\sim}{v}_t, \quad \underset{\sim}{v}_t \sim N(0, \Sigma_v), \tag{19}$$

where $\underset{\sim}{v}_t$ are measurement errors. These errors are zero for consumption and the price level but may be non-zero for bond yields. Appendix C shows how the parameter matrices of this state space model relate to the underlying model parameters, which we estimate using Bayesian methods.

3.3 Estimation method

The model has a total of 20 parameters: the two structural parameters (the discount rate ρ and the degree of risk aversion γ), the average growth rates of consumption and the price level (δ_c and δ_p), the six free parameters in the covariance matrix Σ_u, and the ten measurement error variances. To estimate these parameters we employ a random walk Metropolis-Hastings algorithm, the details of which we outline in Appendix D.

Table 1 shows the prior distributions for the model parameters. The final two columns show the hyperparameters describing the relevant distribution. The two structural parameters, ρ and γ, both follow a Gamma distribution, which restricts them to the positive domain. We choose the shape (τ_1) and scale (τ_2) parameters for the prior distribution of ρ so that the median of the annualized discount rate is around 0.45%, with 5% and 95% quantiles of 0% and 3.84%. This offers a relatively diffuse prior over reasonable values for this parameter. Likewise, the shape and scale parameters governing the distribution of γ are chosen so that γ has a median around 2 with 5% and 95% quantiles of 1.25 and 2.89. Our prior thus assigns a relatively large probability to low values of γ, and our goal is to see whether a low estimate of the degree of risk aversion can deliver a sizable term premium. The priors of δ_p and δ_c, are chosen to reflect a prior average annualized inflation rate of 3% and consumption growth rate of 2%, with 5% and 95% quantiles of (0.53%, 5.47%) and (0.36%, 3.64%), respectively.

We use a data-driven prior for the variances of the transitory and permanent error components. We first use an HP filter to decompose the price level and consumption into their trend and stationary components. Call these τ_{hp} and η_{hp}. From these, we estimate sample variances, fitting the filtered trends to a random walk with drift and the filtered stationary components as white noise. Call these covariance matrices $\Sigma_{\epsilon,hp}$ and $\Sigma_{\eta,hp}$. We use these matrices to parameterize our priors for Σ_ϵ and Σ_η, taking a relatively small degrees of freedom to allow for more uncertainty around these prior estimates. Finally, we use diffuse priors for the variances of the measurement errors, $\sigma^2_{v_i}$.

Table 1: Prior distributions

Parameter	Distribution	τ_1	τ_2
ρ	Gamma	0.5	0.5
γ	Gamma	16	0.125
δ_p	Normal	0.75	0.375
δ_c	Normal	0.5	0.25
Σ_ϵ	Inverse Wishart	4	$\Sigma_{\epsilon,hp}$
Σ_η	Inverse Wishart	4	$\Sigma_{\eta,hp}$
$\sigma^2_{v_i}$	Inverse Gamma	0.01	0.01

Note: the final two columns show the parameter values governing the relevant prior distribution. For the Gamma and Inverse Gamma distributions these are the shape and scale parameters, for the Normal distribution these are the mean and standard deviation, and for the Inverse Wishart they are the degrees of freedom and scale matrix.

3.4 Parameter estimates

Table 2 reports quantiles of all model parameters from both prior and posterior distributions.[5] The density of γ, the degree of risk aversion, has a median of 4.86 with 5% and 95% quantiles of 3.97 and 5.91, indicating some precision around this estimate. This estimate is considerably smaller than much of the existing literature, particularly those works relying on recursive preferences. For example, in models proposed by Tanaka (2024) and Rudebusch and Swanson (2012), the relative risk aversion coefficient is 55 and 110, respectively. In Piazzesi and Schneider (2007), the coefficient is 59 in the benchmark model and needs to increase to 85 in order to match the average slope of the yield curve observed in the data. Other examples of models with recursive preferences that rely on a very large coefficient of risk aversion include van Binsbergen et al. (2012) (66) and Kliem and Meyer-Gohde (2022)

[5]Figure A.3 shows prior and posterior densities of the two structural parameters. We multiply ρ by 4 so it has the interpretation as the annualized discount rate. Both posterior densities are considerably different from their respective prior distributions, indicating that the data are informative about these parameters.

(90). Notable exceptions are Andreasen and Jørgensen (2020) who propose a utility kernel for Epstein-Zin preferences and find that a low relative risk aversion coefficient of 5 is able to match both the equity and bond premium and Creal and Wu (2020) who estimate a value of 1.7 in a model with Epstein-Zin preferences with a stochastic rate of time preference.

Likewise, we find relatively low values for the discount rate. The median annualized discount rate is only 0.05 (i.e., 5 basis points) with 5% and 95% quantiles of 0 and 0.39. This implies a median discount factor of $\beta = e^{-0.05/100} = 0.999$. Creal and Wu (2020) estimate a value of $\beta = 1.002$, similar to the calibrated value of $\beta = 1.004$ from Piazzesi and Schneider (2007) and the estimated value of $\beta = 0.997$ of van Binsbergen et al. (2012), all of which imply a very low discount rate. Herbst and Schorfheide (2016) estimate a small-scale New Keynesian model by Bayesian methods and find a discount rate of 0.42, somewhat larger than our estimate of 4ρ.

Table 2: Estimated parameter values

Parameter	Posterior	Prior	Description
4ρ	0.05	0.45	Annualized discount rate.
	(0.00, 0.39)	(0.00, 3.84)	
γ	4.86	1.96	Risk aversion parameter.
	(3.97, 5.91)	(1.25, 2.89)	
$4\delta_p$	4.97	3.0	Annualized average growth rate of the price
	(4.72, 5.21)	(0.53, 5.47)	level.
$4\delta_c$	0.35	2.0	Annualized average growth rate of
	(0.27, 0.42)	(0.36, 3.64)	consumption.
$\sigma^2_{\epsilon_p}$	0.69	0.13	Variance of the errors of the permanent
	(0.56, 0.87)	(0.04, 0.86)	component of the price level.
$\sigma^2_{\epsilon_c}$	1.36	0.03	Variance of the errors of the permanent
	(1.12, 1.64)	(0.01, 0.17)	component of consumption.
$\sigma_{\epsilon_p \epsilon_c}$	-0.72	0.0	Covariance between errors of the permanent
	(-0.92, -0.56)	(-0.12, 0.12)	components.
$\sigma^2_{\eta_p}$	0.42	0.67	Variance of the temporary component of the
	(0.30, 0.58)	(0.20, 4.52)	price level.
$\sigma^2_{\eta_c}$	0.24	0.92	Variance of the temporary component of
	(0.17, 0.34)	(0.28, 6.20)	consumption.
$\sigma_{\eta_p \eta_c}$	-0.28	0.0	Covariance between errors of the temporary
	(-0.40, -0.21)	(-1.67, 1.67)	components.

Note: Median parameter values from posterior and prior distributions. 5% and 95% quantiles are shown in parentheses.

4 The term structure of interest rates

In this section, we evaluate the ability of the model to match the properties of bond yields across different maturities. The noisy information model successfully produces a yield curve that slopes upward on average, with average yields close to the data for all maturities, and without requiring an unreasonably large estimate of the coefficient of relative risk aversion. We then quantify the relative contribution of real factors, nominal factors, and noisy information for the size of the term premium.

4.1 Bond yields

Table 3 shows average bond yields at the maturities included in our estimation sample, which gives an idea of the average shape of the yield curve. The second column shows statistics for the raw data, the third column shows the same statistics for our noisy information model, and the fourth column shows these statistics for the real bond yields in that model. For our model output, we report the median of these values across all Monte Carlo draws, with the 5% and 95% quantiles shown in parentheses. The final two rows show the average spread between long-run (five or ten year) and short-run (three month) yields. The average ten year to three month spread is estimated to be 85 basis points in the model compared with 98 basis points in the data, indicating that the noisy information model can successfully reproduce the average slope of the yield curve. Most of this spread occurs over the first five years in the model, however, indicating that the slope of the yield curve is somewhat steeper in the model compared with the data.

The model matches average bond yields reasonably closely across all bond maturities, but especially for bonds with maturities in the mid-range of the term structure. By contrast, yields are somewhat too low at both the short and long end of the yield curve. The final column indicates that the real yield curve, given by equation (6), also slopes upward on average and in fact has a larger slope than the nominal yield curve. As we demonstrate below, this can be explained by the negative correlation between the transitory components of consumption and prices, which lowers the nominal term premium relative to the real term premium.

Table 3: Average bond yields

Maturity (months)	Data	Model Nominal	Model Real
3	5.55	5.36	0.03
		(5.16, 5.55)	(-0.50, 0.57)
6	5.63	5.79	0.58
		(5.68, 5.90)	(0.21, 0.97)
12	5.78	6.01	0.86
		(5.92, 6.11)	(0.56, 1.17)
18	5.89	6.09	0.95
		(5.99, 6.18)	(0.67, 1.24)
24	5.98	6.12	1.00
		(6.02, 6.22)	(0.73, 1.28)
36	6.10	6.16	1.04
		(6.06, 6.26)	(0.78, 1.31)
48	6.19	6.18	1.07
		(6.07, 6.28)	(0.81, 1.33)
60	6.27	6.19	1.08
		(6.08, 6.30)	(0.83, 1.34)
84	6.37	6.20	1.10
		(6.09, 6.31)	(0.85, 1.36)
120	6.53	6.21	1.11
		(6.10, 6.32)	(0.86, 1.36)
5 yr. - 3 mth. spread	0.72	0.83	1.04
		(0.62, 1.06)	(0.72, 1.38)
10 yr. - 3 mth. spread	0.98	0.85	1.07
		(0.63, 1.08)	(0.74, 1.42)

Note: Column 2 shows average bond yields in the US over the sample period 1961–2007. Columns 3 and 4 show the median model estimates of the nominal and real yield curves. 90% Bayesian credible intervals in parentheses.

We now turn to assess the model's ability to match the dynamics of bond yields. Figure 1 shows a comparison of yields on bonds maturing in three months and five years in the data (grey line) with those predicted by the model (black line) over the 1962–2007 period. The model predictions track the observable three-month yields quite closely over the full sample period. Through the 1970s bond yields spike twice, both the result of higher inflation stemming from oil price shocks. The first of these episodes is relatively short-lived, and the model over-predicts short-run yields in this period. The predicted bond yields track the level of those in the data much more closely during the second oil price shock, as well as the disinflation until around 1982.

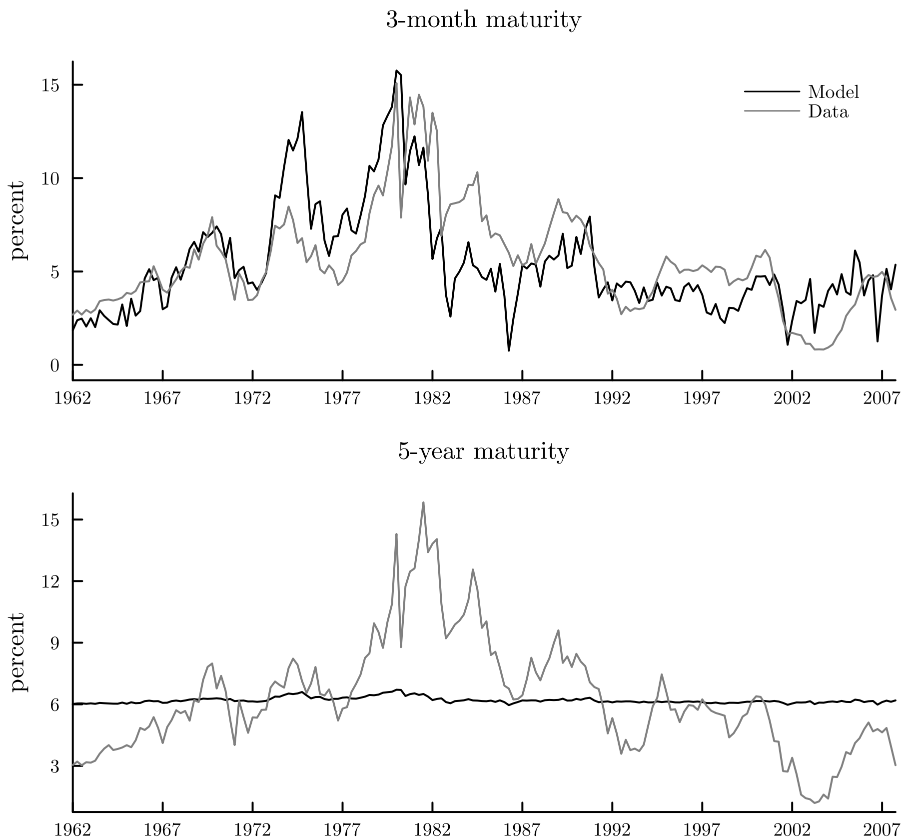

Figure 1: Bond yields

Note: Grey lines show zero-coupon bond yields using data from Liu and Wu (2021) at one year (top panel) and five year (bottom panel) maturities. Black lines are the estimated bond yields from the model at the same maturities, calculated according to (15).

The predicted five-year bond yields continue to match the dynamics seen in the data—the correlation between the two series is 0.64—but are considerably less variable.[6] This finding is consistent with the excess volatility puzzle documented in Shiller (1979) and Piazzesi and Schneider (2007) who show that predicted long-run yields do not capture the volatility

[6]Figure A.4 in the appendix shows that the dynamics in 5-year bond yields is matched quite closely once the yields are normalized by their respective mean and standard deviation.

present in their empirical estimates. Because the term premium is constant in our model, the dynamics of long-run bond yields are explained by the average of the predicted, future short-run bond yields. While predicted short-run yields are volatile enough to match the data, they are not persistent enough to generate the predicted volatility of the long-run yields. It is worth noting that the model correctly matches the sign of the 10 year - 3 month spread 90% of the time over the period June 1961 to December 2007. The term spread is positive 74% of the time in the model, compared with 90% of the time in the data.

4.2 Term premium decomposition

Next, we quantify the relative contribution of real factors, nominal factors, the covariance between these factors, and noisy information for the size of the term premium. The term premium can be rewritten as:[7]

$$0.5\frac{m-1}{m}\left(\underbrace{\gamma^2\sigma^2_{\eta_c}}_{\text{Real}} + \underbrace{\sigma^2_{\eta_p}}_{\text{Nominal}} + \underbrace{\gamma\sigma^2_{\eta_p\eta_c}}_{\text{Covariance}} + \underbrace{[1\ \gamma\ 1\ \gamma]T(\Sigma_\alpha - \Sigma_u)T'[1\ \gamma\ 1\ \gamma]'}_{\text{Noisy information}}\right). \tag{20}$$

The first three terms in the parentheses show the contribution of the variability of the transitory components of consumption, prices, and their covariance, while the final term shows the contribution of noisy information. We see that maturity m affects these terms equally so that the relative contribution of these terms does not depend on bond maturity.

Table 4 shows the relative contributions of each of these factors. We see that real factors explain the largest fraction, while nominal factors on their own explain only 7% of the term premium. Because households want to smooth consumption over time, short-run fluctuations in consumption raise the term premium. The covariance between the transitory components of consumption and prices is negative so that this term actually puts downward pressure on the term premium. This explains why the slope of the real term structure was found to be larger than the slope of the nominal term structure in Table 3 above. It is worth

[7] See appendix for calculations.

noting that in most models with habit formation, this negative correlation between prices and consumption is why the nominal yield curve slopes upwards, whereas we find a negative correlation decreases the size of the term premium, making the yield curve flatter. In our model, a positive correlation between the transitory components of consumption and prices raises short-run uncertainty so that short-term bonds trade at a premium to hedge against risk. Our model is able to reproduce a sizeable term premium on a scale close to what is found in the data even with the negative correlation estimated over the sample period.

Table 4: Term premium decomposition

Factor	Share (%)
Real	96
	(84, 111)
Nominal	7
	(5, 12)
Covariance	-47
	(-66, -34)
Noisy information	44
	(43, 45)

Note: Contribution of each factor to the term premium according to equation (20). 90% Bayesian credible intervals in parentheses.

Taken together, however, economic activity variables explain 96% + 7% - 47% = 56% of the term premium, with the remaining 44% explained by noisy information. This illustrates the important role of noisy information to match the magnitude of the term premium observed in the data. While a full-information model based on the multivariate unobserved components model predicts a positive term premium, the predicted magnitude of the term premium is only about half the size compared to that of a noisy information model.

Finally, we note from Table 2 that there is greater variability in the transitory component of prices than consumption (σ_{η_p} is larger than σ_{η_c}). From equation (20) this indicates that the importance of real activity variables in determining the term premium is not because there is more variability on the real side of the economy. Rather, it is the coefficient of relative risk aversion which amplifies the importance of real activity, relative to nominal effects.

4.3 Alternative specifications

We examine the sensitivity of our findings to two alternative specifications, with the main results shown in Tables A.6 and A.7 found in Appendix E. First, we re-estimate the model over a larger sample from 1961 until 2019. The shorter sample of our benchmark model avoids the zero lower bound, when interest rates remained very low for an extended period of time and unconventional monetary policy actions were undertaken with the specific goal of reducing the spread between short- and long-run interest rates. Our reasoning for excluding this period from our benchmark is twofold. First, it eases comparison with the related literature, much of which also excludes this episode. Second, lower interest rate spreads during this time suggest a lower term premium, which may be easier for the model to match. Table A.6 compares average bond yields in the raw data with model estimates across different maturities. Yields are indeed lower across all maturities, but the performance of the model is overall very similar to the benchmark model. The estimated spread between the 3-month and 10-year yields is 80 basis points compared with 95 basis points in the data.

Next, we extend the model to allow for AR(2) dynamics in the transitory components of both output and the price level. As mentioned in Section 2, the positive sign on the term premium follows from the negative serial correlation in consumption growth and inflation. For the more general AR(2) process, serial correlation can be positive or negative depending on the values of the autoregressive coefficients and the lag in the autocorrelation function. Hence, the term premium can also be positive or negative depending on the estimated parameters and may even change sign at different maturities. Table A.7 shows the results for this alternative specification along with results for the benchmark model for easy comparison. The model continues to match yields well for maturities up to five years; the spread between five-year and three-month bonds is 48 basis points, matching about 2/3 of the spread observed in the data. At longer maturities, however, the term spread begins to decline and eventually turns negative. Campbell (1986) shows that both positive and negative term premia are possible for models with more general dynamics, which explains this finding. Notably, the coefficient of risk aversion in this model actually declines to 2.59.

5 The degree of noisy information

We now turn to the degree of information frictions. In Section 2 we showed that noisy information has two effects on the model: it increases the size of the term premium for given values of the model parameters and it introduces a backward-looking component to the household's expectations of the unobserved components of consumption and prices, which evolves slowly in response to new information. Section 4 showed that noisy information was key for understanding the term structure of interest rates as it accounts for 44% of the scale of the term premium. We now examine the scale of these frictions themselves and how they influence the household's expectations.

To summarize the degree of information rigidity in consumption and prices, we estimate the regression proposed by Coibion and Gorodnichenko (2015), regressing the household's forecast errors on their forecast revisions. Let y_{t+h} denote a macroeconomic variable y at horizon $t+h$, $F_t y_{t+h}$ the h-quarter ahead forecast of variable y made in period t, and $F_{t-1}y_{t+h}$ the forecast made in quarter $t-1$. We estimate the following relationship between h-quarter-ahead forecast errors and forecast revisions:

$$y_{t+h} - F_t y_{t+h} = \beta_0 + \beta_1(F_t y_{t+h} - F_{t-1}y_{t+h}) + error_t \tag{21}$$

for the log of the price level, p, and the log of real consumption, c. Positive values of β_1 can be taken as evidence of information frictions, and larger values are associated with a higher degree of information rigidity. Table 5 presents the estimates of β_1 for horizon

$h = 3$ predicted by our model and using historical forecasts of the CPI and real consumption from the Survey of Professional Forecasters (SPF) over the 1981q4–2019q4 period. Bayesian credible intervals are in parentheses.

Table 5: Test of Information Rigidities

	p	c
Model	2.84	0.92
	(2.74, 2.97)	(0.88, 0.98)
SPF data	1.89	0.23
	(0.59, 3.20)	(-0.08, 0.54)

The estimates in Table 5 provide strong evidence in favour of information rigidities. While this is not surprising given the structure of our model, the results are consistent with what is found using professional forecasts data. In comparison, Coibion and Gorodnichenko (2015) estimate this relationship using SPF forecasts of US variables over the 1968–2014 period. They estimate $\hat{\beta}_1 = 1.141$ for inflation at $h = 3$ and about 0.25 for real consumption. We note that the model's predictions of the household's forecasts are based only on data for consumption, prices, and bond yields, whereas those authors estimate their regressions using forecasts taken from the Survey of Professional Forecasters. Even without survey data, our model estimates of information rigidities are qualitatively similar to what is found in the data.

Table 5 also indicates that information frictions are much larger for prices than for consumption. Recall from the discussion in Section 2 that, without the transitory components, consumption and prices would be perfect signals of their underlying trends. As the variances of the transitory components increase, it becomes harder for the household to distinguish the signal from the noise, so that larger variances of the transitory components result in more information rigidity. From Table 2, we see that $\sigma^2_{\eta_p}$ is much larger than $\sigma^2_{\eta_c}$—both in absolute terms as well as relative to the variance of the respective permanent component—which explains why we find stronger evidence of information rigidities in prices than consumption.

Finally, an important difference between the dynamics of bond yields in the full- and partial-information models is the discrepancy between the transitory components of prices and consumption and the household's filtered estimates of these components. Because the household's estimates enter as state variables in our model (recall equation (17)) we can compare the smoothed estimates of these components. Figure 2 shows the smoothed estimated temporary components (black lines) as well as the household's estimates of the temporary components (grey lines). Each of these series are model output, but they differ because the econometrician's information set includes the full sample of data (i.e., the entire time period), whereas the household's information set includes information only until period t. In other words, the full sample of bond yields, consumption, and prices help to estimate the household's Kalman filtered forecasts of the state of the economy. When the household's expectations deviate substantially from the econometrician's, providing the household with additional sample information would have a large effect on their expectations.

The difference between the black and grey lines in Figure 2 gives an idea of the cost of partial information. As discussed above, under partial information, expectations will be slow to adjust to new information. We see that for both variables the household's expectations lag the underlying components. The biggest movements in these series occur around the late 1970s. Because these are temporary components, that indicates that a substantial amount of the increase in inflation and decline in consumption around this time were perceived to be temporary changes to economic activity. The differences between the black and grey lines during this time, however, indicate that markets over-evaluated the short-run nature of these shocks. We see that the black lines are much closer to zero, indicating that the best forecast using all available data is that these shocks were instead explained by long-run factors. This discrepancy is explained by information frictions.

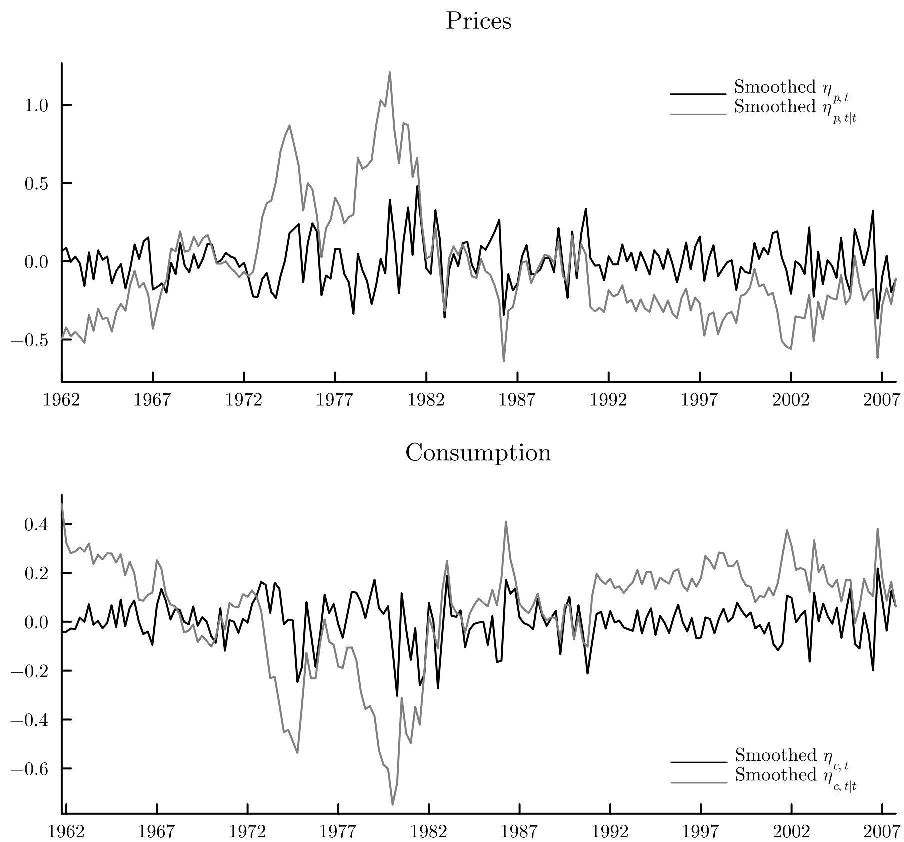

Figure 2: Temporary components

Note: Black lines show the estimates of the temporary components of consumption and the price level using the Kalman smoother. Grey lines show smoothed estimates of the household's filtered period-t forecasts of these components.

6 Conclusion

We study the term structure of interest rates in an endowment economy with noisy information. The endowment and price level each consist of temporary and permanent components, but the household observes only the total endowment and overall price level. To forecast future economic conditions, and hence price bonds of various maturities, the household must first solve a signal extraction problem using the Kalman filter. This results in additional uncertainty, raising the household's forecast variances and leading to a larger term premium compared with a full-information version of the model.

We then estimate the model using Bayesian methods and US data between 1961–2007. The average spread between ten-year and three-month bonds in our partial-information model is estimated to be 85 basis points compared with 98 basis points in the data, a significant improvement relative to a comparable full-information model. Further, the model does not require a very large coefficient of relative risk aversion, which we estimate to be 4.86. Without the key feature of noisy information, the estimated interest rate spread would be only half the size of our benchmark estimates. Among economic activity variables, we show that real factors are much more important than nominal factors, which have only a small impact on the term premium. This is not because there is more variability on the real side of the economy, but rather because this variability is amplified by the coefficient of relative risk aversion.

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A Deriving real bond prices

Taking logs of (3) and using $\exp(c_{t+m}) = C_{t+m}$ gives:

$$\log q^r_{t,m} = -\rho m + \gamma c_t + \log(E_t[\exp(c_{t+m})^{-\gamma}]). \tag{A.22}$$

See that:

$$E_t[c_{t+m}] = m\delta_c + \tau_{c,t|t},$$
$$E_t[(c_{t+m} - \hat{c}_{t+m})^2] = \sigma^2_{\tau_c} + (m-1)\sigma^2_{\epsilon_c} + \sigma^2_{\eta_c}.$$

Using the properties of a log-normal distribution we then have:

$$E_t[\exp(-\gamma c_{t+m})] = \exp(-\gamma m\delta_c - \gamma\tau_{c,t|t} + 0.5\gamma^2(\sigma^2_{\tau_c} + (m-1)\sigma^2_{\epsilon_c} + \sigma^2_{\eta_c})), \tag{A.23}$$

so that bond yields are:

$$yld^r_{t,m} = \rho - \frac{1}{m}\gamma c_t + \gamma\delta_c + \frac{1}{m}\gamma\tau_{c,t|t} - 0.5\frac{1}{m}\gamma^2(\sigma^2_{\tau_c} + (m-1)\sigma^2_{\epsilon_c} + \sigma^2_{\eta_c}). \tag{A.24}$$

Using $c_t = \tau_{c,t|t} + \eta_{c,t|t}$ gives (6). The spread between yields on m- and one-period bonds is then:

$$yld^r_{t,m} - yld^r_{t,1} = (1 - 1/m)\gamma\eta_{c,t|t} + 0.5(1 - 1/m)\gamma^2(\sigma^2_{\tau_c} + \sigma^2_{\eta_c} - \sigma^2_{\epsilon_c}). \tag{A.25}$$

Using $\sigma^2_{\tau_c} - \sigma^2_{\epsilon_c} = (1/\sigma^2_{\eta_c} + 1/\sigma^2_{\epsilon_c})^{-1}$ from the Ricatti equation gives:

$$yld^r_{t,m} - yld^1_{t,1} = (1 - 1/m)\gamma\eta_{c,t|t} + 0.5(1 - 1/m)\gamma^2(\sigma^2_{\eta_c} + (1/\sigma^2_{\eta_c} + 1/\sigma^2_{\epsilon_c})^{-1}),$$

which matches (7).

B Deriving the term premium

In general, we can express the yield on a bond maturing in m periods as the average of current and expected future one-period bond yields and a term premium:

$$yld_{t,m} = \frac{1}{m}\sum_{j=0}^{m-1} E_t[yld_{t+j,1}] + tp_{t,m}. \tag{A.26}$$

From (6), expected future one-period yields are:

$$E_t[yld_{t+j,1}] = \rho + \delta_p + \gamma\delta_c + [1\ \gamma]\Lambda(T - I)\alpha_{t+j|t} - 0.5[1\ \gamma]\Lambda M_{t+j+1|t+j}\Lambda'[1\ \gamma]'.$$

From the state equations we see that:

$$\alpha_{t+j|t} = j \times \mu + T\alpha_{t|t},$$

which uses $T^j = T$ since there is no persistence in the temporary components. See also that $\Lambda(T-I)\mu = 0$ so that expected future bond yields are:

$$E_t[yld_{t+j,1}] = \rho + \delta_p + \gamma\delta_c - [1\ \gamma]\Lambda(T-I)T\alpha_{t|t} - 0.5[1\ \gamma]\Lambda\Sigma_\alpha\Lambda'[1\ \gamma]', \tag{A.27}$$

which uses $M_{t+j+1|t+j} = \Sigma_\alpha$ when the Kalman filter has converged.

Substituting this result into (A.26) and rearranging for the term premium gives:

$$\begin{aligned}
tp_{t,m} &= yld_{t,m} - \frac{1}{m}yld_{t,1} - \frac{1}{m}\sum_{j=1}^{m-1} E_t[yld_{t+j,1}] \\
&= \frac{1}{m}[1\ \gamma]\Lambda T^m\alpha_{t|t} - \frac{1}{m}[1\ \gamma]\Lambda T\alpha_{t|t} - 1/m\sum_{j=1}^{m-1}[1\ \gamma]\Lambda(T-I)T\alpha_{t|t} \\
&- \frac{0.5}{m}[1\ \gamma]\Lambda M_{t+m|t}\Lambda'[1\ \gamma]' + \frac{0.5(m-1)}{m}[1\ \gamma]\Lambda\Sigma_\alpha\Lambda'[1\ \gamma]'.
\end{aligned}$$

This reduces to:

$$tp_{t,m} = 0.5[1\ \gamma]\Lambda(\Sigma_\alpha - 1/mM_{t+m|t})\Lambda'[1\ \gamma]', \tag{A.28}$$

which is time invariant when the Kalman filter has converged and $M_{t+m|t}$ is a constant that depends only on m:

$$M_{t+m|t} = T^{m-1}\Sigma_\alpha T^{m-1'} + \sum_{j=0}^{m-2} T^j\Sigma_u T^{j'}. \tag{A.29}$$

Since $\Sigma_\alpha = M_{t+1|t}$, we see that the term premium matches the final term of the interest rate spread in equation (16).

By comparison, in the full-information model, we have:

$$\Sigma_\alpha^{FI} = \Sigma_u, \tag{A.30}$$

$$M_{t+m|t}^{FI} = \sum_{j=0}^{m-1} T^j\Sigma_u T^{j'}. \tag{A.31}$$

To decompose the different factors behind the term premium, first add and subtract the terms Σ_u and $1/m\sum_{j=0}^{m-1} T^j\Sigma_u T^{j'}$ to the expression in parentheses, which allows us to rewrite the term premium as:

$$\begin{aligned}
tp_{t,m} &= 0.5[1\ \gamma]\Lambda\left(\Sigma_u - 1/m\sum_{j=0}^{m-1} T^j\Sigma_u T^{j'}\right)\Lambda'[1\ \gamma]' \\
&+ 0.5[1\ \gamma]\Lambda\left((\Sigma_\alpha - \Sigma_u) - 1/m(M_{t+m|t} - \sum_{j=0}^{m-1} T^j\Sigma_u T^{j'})\right)\Lambda'[1\ \gamma]'.
\end{aligned} \tag{A.32}$$

The first term shows the impact of macroeconomic variables on the term premium while the second is due to information frictions. Using $[1\ \gamma]\Lambda = [1\ \gamma\ 1\ \gamma]$ and $T^j = T\ \forall\ j > 0$ we have:

$$\Sigma_u - T\Sigma_u T' = \begin{bmatrix} 0_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 2} & \Sigma_\eta \end{bmatrix},$$

$$1/m(M_{t+m|t} - \sum_{j=0}^{m-1} T^j \Sigma_u T^{j'}) = 1/mT(\Sigma_\alpha - \Sigma_u)T',$$

which gives:

$$\begin{aligned} tp_{t,m} = {} & 0.5\frac{(m-1)}{m}[1\ \gamma\ 1\ \gamma] \begin{bmatrix} 0_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 2} & \Sigma_\eta \end{bmatrix} [1\ \gamma\ 1\ \gamma]' \\ & + 0.5[1\ \gamma\ 1\ \gamma]((\Sigma_\alpha - \Sigma_u) - 1/mT(\Sigma_\alpha - \Sigma_u)T')[1\ \gamma\ 1\ \gamma]'. \end{aligned} \tag{A.33}$$

Finally, because there is no persistence in the transitory components and zero correlation between the permanent and transitory errors, the lower-right quadrants of Σ_α and Σ_u will be the same and we have:

$$\Sigma_\alpha - \Sigma_u = T(\Sigma_\alpha - \Sigma_u)T',$$

so that the term premium can be written as:

$$tp_{t,m} = 0.5\frac{(m-1)}{m}[1\ \gamma\ 1\ \gamma] \begin{bmatrix} 0 & 0 \\ 0 & \Sigma_\eta \end{bmatrix} [1\ \gamma\ 1\ \gamma]' + 0.5\frac{(m-1)}{m}[1\ \gamma\ 1\ \gamma]T(\Sigma_\alpha - \Sigma_u)T'[1\ \gamma\ 1\ \gamma]', \tag{A.34}$$

where the first term is due to the variability of macroeconomic conditions and the second term arises because of information frictions. Straightforward multiplication shows that this matches equation (20) from the main text.

C State space model

Under the assumption that the household's Kalman filter has converged, their state variables evolve as:

$$\alpha_{t|t} = \mu + (I - \Sigma_\alpha \Lambda' F^{-1}\Lambda)T\alpha_{t-1|t-1} + \Sigma_\alpha \Lambda' F^{-1}\Lambda T\alpha_{t-1} + \Sigma_\alpha \Lambda' F^{-1}\Lambda u_t, \tag{A.35}$$

where $F = \Lambda\Sigma_\alpha\Lambda'$ and $\Sigma_\alpha = M_{t|t-1}$ is the solution to the Riccati equation:

$$\Sigma_\alpha - T\Sigma_\alpha T' + T\Sigma_\alpha\Lambda'(\Lambda\Sigma_\alpha\Lambda')^{-1}\Lambda\Sigma_\alpha T' - \Sigma_u = 0. \tag{A.36}$$

The parameter matrices from the state equation (18) are then:

$$\underset{\sim}{\mu} = [\delta_p,\ \delta_c,\ 0,\ 0,\ \delta_p,\ \delta_c,\ 0,\ 0,\ 0,\ 0]' \tag{A.37}$$

$$\underset{\sim}{T} = \begin{bmatrix} T & 0 \\ \Sigma_\alpha \Lambda' F^{-1} \Lambda T & (I - \Sigma_\alpha \Lambda' F^{-1} \Lambda) T \end{bmatrix} \tag{A.38}$$

$$\underset{\sim}{R} = \begin{bmatrix} I \\ \Sigma_\alpha \Lambda' F^{-1} \Lambda \end{bmatrix} \tag{A.39}$$

The first two elements of x_t are prices and consumption so that the first two rows of $\underset{\sim}{d}$ are both 0 and the first two rows of $\underset{\sim}{\Lambda}$ are:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0_{1\times 4} \\ 0 & 1 & 0 & 1 & 0_{1\times 4} \end{bmatrix} \tag{A.40}$$

The remaining elements of x_t are bond yields and the corresponding rows of $\underset{\sim}{d}$ and $\underset{\sim}{\Lambda}$ depend on the bond maturity. For $\underset{\sim}{\Lambda}$, the row entry is:

$$\begin{bmatrix} -1/m & -\gamma/m & -1/m & -\gamma/m & 1/m[1\ \ \gamma]\Lambda T^m \end{bmatrix} \tag{A.41}$$

For $\underset{\sim}{d}$, the row entry is:

$$\rho + [1\ \ \gamma]\Lambda\mu - 0.5m^{-1}[1\ \ \gamma]\Lambda \left(\frac{M_{t+m|m}}{100} \right) \Lambda'[1\ \ \gamma]' \tag{A.42}$$

where:

$$M_{t+m|t} = T M_{t+m-1|t} T' + \Sigma_u, \tag{A.43}$$

which has a time-invariant solution when the household's Kalman filter has converged using $M_{t|t-1} = \Sigma_\alpha$. Notice that the covariance matrix $M_{t+m|m}$ is divided by 100 in (A.42) because we rescale the data from decimal to percentage points. As we explain below, this rescaling improves the efficiency of the Monte Carlo estimator because fewer draws are rejected for fallowing outside the range of the prior distributions. Finally, the covariance matrix of the measurement errors Σ_v is assumed to be diagonal with the first two elements set to zero, because prices and consumption are decomposed into the sum of their temporary and permanent components, both of which are state variables.

D Estimation details

Let θ be the vector of model parameters and Y the observable data, we aim to sample from the posterior distribution of the model parameters:

$$p(\theta|Y) \propto p(Y|\theta)p(\theta), \tag{A.44}$$

where $p(Y|\theta)$ is the density of the likelihood and $p(\theta)$ the prior distribution of the parameters. With our assumption of normal errors, the likelihood can be evaluated using the

Kalman filter. But sampling from the posterior distribution is challenging because the prior distributions of the structural model parameters result in a posterior that does not follow a standard distribution. This problem is typical of the estimation of structural macroeconomic models, and we follow the standard solution which is to employ a random walk Metropolis-Hastings algorithm. We sample candidate draws from a Normal proposal distribution and accept these draws with a probability determined by the value of the likelihood function at the candidate draw (see Herbst and Schorfheide (2016) for a detailed review of this method). Because of the moderate number of parameters, we split up the parameters into four blocks: the first block contains the two structural parameters (ρ and γ), the second block contains the average growth rates for consumption and prices (δ_c and δ_p), the third block contains the variances of the shocks to the transitory and permanent components, and the fourth block contains the measurement error variances. The algorithm then proceeds as follows:

0. Choose initial mean (θ_0) and variance ($c^2\Sigma$) for the proposal distribution.
1. For block b, draw a candidate $\tilde{\theta}_b$ from the distribution $N(\theta_{b,s-1}, c^2\Sigma_b)$.
2. Accept the draw $\tilde{\theta}_b$ with probability:

$$\min\left\{1, \frac{p([\tilde{\theta}_{<b,s},\ \tilde{\theta}_b,\ \tilde{\theta}_{>b,s-1}]|Y)}{p([\tilde{\theta}_{<b,s},\ \tilde{\theta}_{b,s-1},\ \tilde{\theta}_{>b,s-1}]|Y)}\right\},$$

   otherwise set $\theta_{b,s} = \theta_{b,s-1}$.
3. Repeat steps 1 and 2 for each block.
4. Repeat steps 1 to 3 for N draws.

We set $N = 500,000$ and discard the first $N/2$ draws to allow the algorithm to converge. We initialize the algorithm by setting θ_0 to the posterior median of ρ and γ and the posterior mode of the prior distribution of the remaining parameters. To parameterize Σ, we run a preliminary Markov Chain of 500,000 samples using an identity matrix for Σ with $c = 0.05$. We then re-estimate the model setting Σ to the covariance matrix of θ from the initial run and increasing c to achieve an acceptance rate of around 30%.

To ensure that the proposal distribution gives a positive definite covariance matrix Σ_u we reparameterize this matrix using the decomposition $L\Sigma_u L' = D$ where D is a diagonal matrix and L is a lower-diagonal matrix with ones on the main diagonal. Chan and Jeliazkov (2009) show that an Inverse Gamma prior distribution for the diagonal elements of D and a Normal prior distribution for the lower diagonal elements of L implies a Wishart prior distribution for Σ_u^{-1}. Chan (2022) further shows the choice of priors for these new matricies that correspond with the priors for Σ_ϵ and Σ_η listed in Table 1.

A final challenge is that, because we use a Normal distribution as the proposal distribution, it may propose draws for some parameters that fall outside the domains of their prior distributions. This is primarily an issue for ρ and the measurement errors when the bond yields are in decimal percent because these parameters will be very close to zero. In principal, these draws can simply be skipped, but if there are many such draws the algorithm will be inefficient. To address this, we multiply the data by 100 to convert to percentage points.

E Additional tables and figures

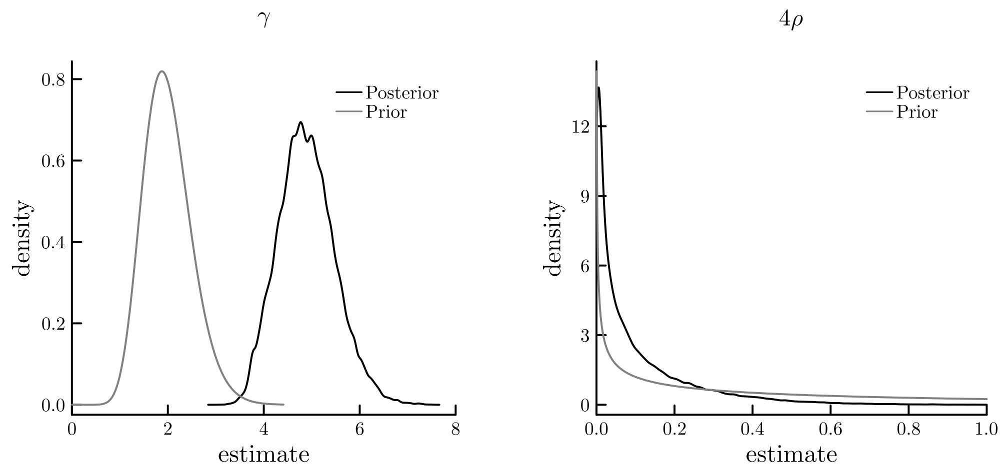

Figure A.3: Kernel densities of structural parameters

Note: Kernel densities of the prior (grey) and posterior (black) distributions of the discount factor ρ and degree of risk aversion γ based on 250,000 draws using a random walk Metropolis-Hastings algorithm.

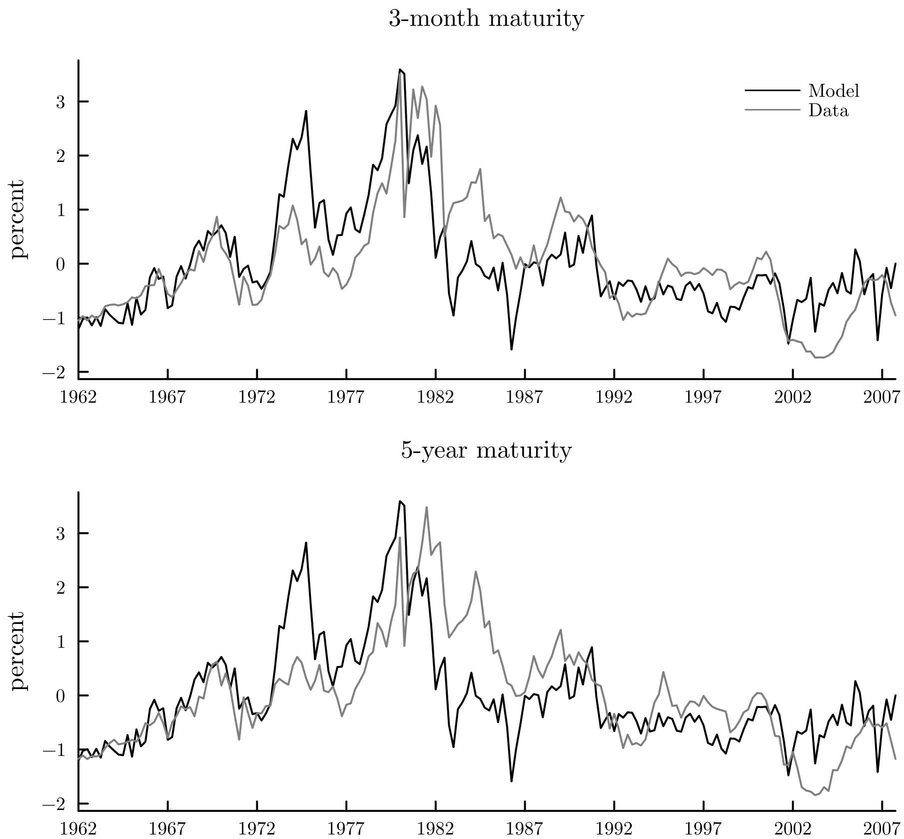

Figure A.4: Standardized bond yields

Note: Grey lines show zero-coupon bond yields using data from Liu and Wu (2021) at one year (top panel) and five year (bottom panel) maturities normalized using their respective mean and standard deviation. Black lines are the estimated bond yields from the model at the same maturities, calculated according to (15), normalized using their respective mean and standard deviation.

Table A.6: Average bond yields in the extended sample

Maturity (months)	Data	Model Nominal	Model Real
3	4.52	4.33	-0.63
		(4.12, 4.55)	(-1.15, -0.07)
6	4.59	4.74	-0.09
		(4.63, 4.86)	(-0.43, 0.28)
12	4.72	4.94	0.18
		(4.85, 5.04)	(-0.08, 0.46)
18	4.83	5.01	0.27
		(4.91, 5.11)	(0.03, 0.53)
24	4.91	5.04	0.31
		(4.94, 5.15)	(0.09, 0.56)
36	5.02	5.08	0.36
		(4.97, 5.19)	(0.14, 0.59)
48	5.11	5.10	0.38
		(4.98, 5.21)	(0.17, 0.61)
60	5.18	5.11	0.39
		(4.99, 5.22)	(0.19, 0.62)
84	5.29	5.12	0.41
		(5.00, 5.23)	(0.21, 0.63)
120	5.47	5.13	0.42
		(5.01, 5.24)	(0.22, 0.64)
5 yr. - 3 mth. spread	0.66	0.78	1.02
		(0.52, 1.02)	(0.63, 1.39)
10 yr. - 3 mth. spread	0.95	0.80	1.04
		(0.53, 1.05)	(0.64, 1.43)

Note: Column 2 shows average bond yields in the US over the sample period 1961–2019. Columns 3 and 4 show the median model estimates of the nominal and real yield curves. 90% Bayesian credible intervals in parentheses. The median coefficient of risk aversion is $\gamma = 6.13$ and the median discount rate is $4\rho = 0.06$.

Table A.7: Average nominal bond yields: Alternate specification

Maturity	Data	Benchmark	AR(2)
3	5.55	5.36	5.77
		(5.16, 5.55)	(5.64, 5.89)
6	5.63	5.79	5.83
		(5.68, 5.90)	(5.69, 5.98)
12	5.78	6.01	6.08
		(5.92, 6.11)	(5.98, 6.17)
18	5.89	6.09	6.18
		(5.99, 6.18)	(6.10, 6.27)
24	5.98	6.12	6.23
		(6.02, 6.22)	(6.15, 6.32)
36	6.10	6.16	6.27
		(6.06, 6.26)	(6.18, 6.37)
48	6.19	6.18	6.27
		(6.07, 6.28)	(6.17, 6.37)
60	6.27	6.19	6.24
		(6.08, 6.30)	(6.15, 6.34)
84	6.37	6.20	6.10
		(6.09, 6.31)	(6.00, 6.20)
120	6.53	6.21	5.44
		(6.10, 6.32)	(5.19, 5.66)
5 yr. - 3 mth. spread	0.72	0.83	0.48
		(0.62, 1.06)	(0.36, 0.61)
10 yr. - 3 mth. spread	0.98	0.85	-0.33
		(0.63, 1.08)	(-0.60, -0.07)

Note: Column 2 shows average bond yields in the US over the sample period 1961–2007. Columns 3 and 4 show the median model estimates of the nominal and real yield curves. 90% Bayesian credible intervals in parentheses. The median coefficient of risk aversion is $\gamma = 2.59$ and the median discount rate is $4\rho = 0.05$.