## Monetary policy and the term structure of Inflation expectations with information frictions

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#### Abstract

Inflation expectations play an important role in macroeconomics, influencing the real interest rate, actual inflation, as well as the transmission of monetary policy. In this paper I estimate how expectations of inflation in the United States (taken from the *Survey of Professional Forecasters*) respond to monetary policy shocks from 1992 to 2018 while accounting for the presence of information frictions. I first use the unobserved components model, which decomposes expected inflation at any forecast horizon into expected inflation in the short and long run, to estimate the term structure of inflation expectations. I find that outdated information accounts for approximately 28% of inflation forecasts. Information frictions are important on average but play an especially large role during recessionary periods. I then show that long-run expectations decline after a monetary policy contraction, and the effect is permanent. I also show that monetary policy actions taken throughout the recession beginning in 2008 effectively propped up long-run inflation expectations from 2009 to 2014.

Keywords: Monetary policy, inflation expectations, sticky information, state-space model.

JEL classification: C32, E31, E52.

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## 1 Introduction

Expectations play a foundational role in modern macroeconomics. Agents are forward looking in virtually all models so that their actions, and hence model equilibria, depend on their expectations of future events. Among expectations perhaps none has received greater interest than inflation expectations, which play a role in determining the real interest rate, through the Fisher equation, as well as actual inflation, through the New Keynesian Phillips curve, two key variables for central bankers. The ability of the monetary authority to influence inflation expectations is especially important when interest rates are at their effective lower bound to avoid falling into a liquidity trap. In such a situation the monetary authority may be able to put downward pressure on real interest rates if it is able to increase inflation expectations through unconventional monetary policy operations such as forward guidance and asset purchase programs. An understanding of how inflation expectations respond to changes in monetary policy is hence an important question.

In models with rational expectations, expected inflation responds to monetary policy in a manner consistent with actual inflation. In this scenario, knowledge of how actual inflation responds to monetary policy implies knowledge of how expected inflation will respond to the same policy. But many recent papers have documented that observable expectations have properties inconsistent with full-information rational expectations and consistent with models with information frictions. How these information frictions affect the response of inflation expectations after changes to monetary policy is less well understood.

In this paper I estimate how US inflation expectations, taken from the *Survey of Professional Forecasters*, respond to changes in monetary policy over the period 1992–2018, while allowing for the presence of information frictions in the underlying survey data. The exercise sheds light on the transmission of monetary policy through inflation expectations, the quantitative importance of information frictions, and the interaction of the two. To do this I estimate a term structure model of inflation expectations using the unobserved components model, which provides a mapping from latent model variables to inflation forecasts at

any horizon. This allows the common information in survey forecasts at all horizons to be incorporated in my analysis in a consistent manner.

To account for information frictions, I allow observed expectations to depart from the unobserved components model in a predictable manner, consistent with models where expectations are slow to adjust to new information, as in the sticky information model proposed by Mankiw and Reis (2002). I find that outdated information accounts for 28% of inflation forecasts. The quantitative importance of information frictions also appears to vary over the business cycle, even though the model coefficients are not time-varying; differences between inflation expectations uncovered from models with and without information frictions are especially large during recessionary periods compared with non-recessionary periods.

To estimate the response of the entire term structure of inflation expectations after a monetary policy shock I use SVAR-IV methods proposed by Stock and Watson (2012) and Mertens and Ravn (2013). Following Gertler and Karadi (2015) and Miranda-Agrippino and Ricco (2017) I use the change in three-month federal funds futures contracts after monetary policy announcements as an instrument for exogenous interest rate fluctuations. I find that contractionary monetary policy decreases inflation expectations with a lag of several years, consistent with the empirical evidence that actual inflation falls after contractionary monetary policy shocks but with a lag of several years. This response can be explained by a temporary increase in the short-run component of inflation expectation and a permanent decrease in the long-run component of inflation expectations. For the first several years after a monetary policy shock these two opposite effects cancel each other out, so there is no effect on expectations overall. But, in the long-run, the permanent effect dominates and expectations decline. Historical decompositions show that the overall contribution of monetary policy shocks to changes in inflation expectations is small for short-run expectations but large for long-run expectations. In particular, I show that monetary policy shocks accounted for a substantial amount of the variation in long-run inflation expectations from 2009 to 2014, a period when the Federal Reserve was constrained at the zero lower bound of nominal interest rates.

### 2 Research context

Empirical evidence about the behavior of expectations is important because models which allow for departures from full-information rational expectations make different predictions about the effects of monetary policy. In this paper I consider the effects of information constraints, which can be interpreted as arising from either sticky or noisy information. In the sticky information model, proposed by Mankiw and Reis (2002), aggregate expectations are slow to adjust to new information, which leads to a delayed response of actual inflation to a monetary policy shock, consistent with the results of much of the empirical monetary policy literature. Aggregate expectations are also slow to adjust to new information in the noisy information model proposed by Woodford (2003). While the source of the information frictions differs in the sticky and noisy information models, Coibion and Gorodnichenko (2012) show that they share the common feature that outdated information plays a role in aggregate expectations. For simplicity I interpret the presence of outdated information as arising because of sticky information but acknowledge that alternative interpretations, such as the noisy information model, are possible. I emphasize, however, that the focus of this research is not the source of the information frictions but rather their size and implications.

From an empirical perspective, evidence against rational expectations has primarily focused on the forecasting properties of survey forecasts. Mehra (2002), Adam and Padula (2011), and Patton and Timmermann (2012) show that survey forecasts are biased, inefficient, and have serially correlated forecast errors, behavior that is inconsistent with rational expectations. Coibion and Gorodnichenko (2012, 2015) show that these properties are consistent with models featuring information frictions and can be large, even for professional forecasters. Kiley (2007), Döpke *et al.* (2008), Coibion (2010) and Pfajfar and Roberts (2018) estimate versions of the Phillips curve while allowing for information frictions and also find departures from full-information rational expectations.

In this paper I estimate a term structure model of inflation expectations based on the unobserved components model, while accounting for information frictions. The model provides a snapshot of expected inflation over any given forecast horizon at any point in time as a function of dynamic short- and long-run latent components. Analyzing the responses of these factors after a monetary policy shock allows for a greater understanding of the responses of short- and long-run inflation expectations. Since the model can be matched to inflation expectations between any two forecast horizons it can accommodate both fixed-horizon and fixed-window forecasts, following the approach used by Aruoba (2019). Once in its state space representation the model parameters, including the information friction parameters, can be estimated by maximum likelihood using numerical methods.

The unobserved components model is a natural candidate to estimate the term structure of inflation expectations because it is known to match the properties of actual inflation quite well. Stock and Watson (2007) show that the unobserved components model can be used to decompose inflation into a permanent random-walk component and a transitory meanreverting component. Cogley *et al.* (2010) use a version of this model to measure changes in the persistence of the inflation gap—the difference between inflation and its long run trend—and Cogley and Sargent (2015) document changes in uncertainty of the permanent and transitory components of inflation over a long time series.

Kozicki and Tinsley (2012) apply the unobserved components model to estimate the term structure of inflation expectations, reasoning that, since the model fits actual inflation quite well, it should also fit inflation expectations quite well. Crump *et al.* (2018) extend this model to estimate jointly the term structure of expectations of inflation, output, and interest rates and Maruyama *et al.* (2019) estimate a similar model for expectations in Japan. Mertens (2016) uses the unobserved components model to measure the trend component of inflation expectations and Nason and Smith (2020) extend the model to account for sticky information in the survey forecasts.

Other term structure models have also been fit to expected inflation. Mehrotra and Yetman (2014) estimate a term structure model where expectations converge from the current level of inflation to long-run expectations. Aruoba (2019) and Kapetanios *et al.* (2016) have instead applied the Nelson and Siegel (1987) model to inflation expectations. The Nelson– Siegel model is a prominent model for the term structure of bond yields, popularized by Diebold and Li (2006) and Diebold *et al.* (2006). One drawback is that the model is not consistent across time, as shown by Björk and Christensen (1999) and Filipović (1999), and hence the expectations uncovered from the model cannot coincide with full-information rational expectations. This is an issue when trying to estimate the size of information frictions which requires a model to stand in for full-information rational expectations, as I show in detail in Section 3. Krippner (2006) shows how the Nelson-Siegel model can be modified to achieve consistency with restrictions on the dynamics of the latent factors but it is not clear how this can be extended to allow the factors to jointly evolve with observable macroeconomic variables, a necessary component to estimate the impact of monetary policy on inflation expectations.

Once estimated, I use the model to analyze the impact of monetary policy on inflation expectations. Because the model uses data at all available forecast horizons this provides a complete analysis of the response of inflation expectations generally. The conventional view of monetary policy is that higher interest rates lower inflation after a lag of several years. The results of Romer and Romer (2004) and Gertler and Karadi (2015), for example, support this view. If expectations are consistent with actual inflation, as they are in the vast majority of theoretical models, they should respond in the same manner. Further, if the central bank has a credible inflation target then long-run expectations should be stable at that target and not respond to short-run policy moves. This provides a set of testable predictions about the response of expectations after a monetary policy shock in both the short and long run.

I also consider a contrarian view which propose the opposite: higher interest rates in-

crease inflation expectations. One explanation for this behavior is the signalling channel of monetary policy, where policy changes are informative public signals about the state of the economy. Romer and Romer (2000) show that forecasts from the Federal Reserve's Greenbook outperform those of private forecasters and provide some evidence that forecasters infer this information from policy moves to update their forecasts. Nakamura and Steinsson (2018) find that forecasts of output respond contemporaneously and positively to contractionary monetary policy shocks. This finding motivates their model in which policy changes are informative about the true state of the economy through an information channel. Likewise, Jarociński and Karadi (2020) show that, in many cases, stock prices move in the same direction as interest rate surprises—the opposite direction predicted by standard theory—a finding they use to decompose monetary policy shocks into a standard component and an information component.

Melosi (2016) shows that, in a model where firms have incomplete information similar to the noisy information model, monetary policy shocks amount to a public signal, revealing information about the monetary authority's information set. This leads to a signalling channel where contractionary monetary policy shocks signal positive demand shocks so that agents update their inflation expectations upwards. The signalling channel arises in this model because information is dispersed: agent's receive private noisy signals about the true state of the economy so that the public signal, the monetary policy change, provides additional information about the state of the economy. If agents have full information, the public signal reveals nothing and expectations respond only to the anticipated policy effect. Hence, accounting for information frictions while estimating the term structure of expectations is important in order to properly test for evidence of such an information effect.

Falck *et al.* (forthcoming) provide empirical support for this theory by showing that contractionary monetary policy shocks cause short-run inflation expectations to rise in periods of high-disagreement among forecasters and fall in periods of low-disagreement. They interpret this using the signalling channel of monetary policy with an extension of the Melosi (2016) model, so that the precision of public signals, and hence disagreement among agents, is time-varying. When disagreement among forecasters is high, public signals are highly informative and hence the signalling channel is strong and moves expectations in the same direction as interest rates. By contrast, when disagreement is low, public signals are not as informative, so the dominant effect is the expected policy effect, which moves inflation expectations in the opposite direction as interest rates.

Empirical evidence on the effects of monetary policy on inflation expectations is somewhat mixed and papers tend to focus on either short- or long-run expectations. Many studies of long-run expectations use inflation compensation, measured as the difference between nominal and real bond yields, as a proxy for inflation expectations. Both Hanson and Stein (2015) and Nakamura and Steinsson (2018) find that long-run inflation compensation falls after a monetary policy contraction, but the effect is only significant in the latter study.

Gürkaynak *et al.* (2005a) find that long-run inflation compensation for the United States is responsive to news about a variety of macroeconomic variables as well as monetary policy. Gürkaynak *et al.* (2010) and Gürkaynak *et al.* (2007) show that this effect does not extend to countries with formal inflation targets which they interpret as evidence that inflation targets are effective in stabilizing long-run inflation expectations. Beechey and Wright (2009) also find that long-run inflation compensation declines after a monetary policy tightening but argue that this could instead represent a change to the inflation risk premium rather than inflation expectations.

Distinguishing between the response of expected inflation and inflation risk is a general problem with relying on inflation compensation derived from bond yields as a proxy for inflation expectations, which I avoid by instead using survey data. An additional advantage is that survey data is typically more informative about short-run expectations than financial data so that I can estimate the response of the entire term structure of inflation expectations. Several studies estimate the responsiveness of survey data to changes in the macroeconomy but these typically focus on either short- or long-run expectations. Leduc *et al.* (2007) and Melosi (2016) show that short-run inflation expectations decline after a monetary tightening without the long delay typically seen from actual inflation. Davis (2014) shows that shortrun expectations are less responsive to changes in actual inflation or the price of oil in countries with formal inflation targets but do not consider monetary policy shocks. Finally, Bauer (2015) shows that long-run expectations from surveys do respond to macroeconomic news but do not consider the effects of monetary policy. To my knowledge the research in this paper is the first to estimate the response of the entire term structure of inflation expectations to a monetary policy shock.

Finally, recent works by Claus and Nguyen (2019), Lamla and Vinogradov (2019), Lewis et al. (2019), and Coibion et al. (2020) estimate the effects of monetary policy on household expectations. Most of these papers focus in particular on the ability of unconventional monetary policy to shift household expectations. Coibion et al. (2020), for example, provide experimental evidence of a strong effect of forward guidance on household expectations. Household expectations are of particular interest because of the direct connection to the expectations driving theoretical macroeconomic models. But the term structure component of this data is typically limited, often consisting only of average forecasts at a few forecast horizons. By contrast, data from the Survey of Professional Forecasters contains a rich term structure component, which makes it more appropriate for the application in this paper.

## 3 The unobserved components model

To estimate the effects of monetary policy on inflation expectations at all horizons I estimate a term structure model of inflation expectations based on the unobserved components model, which allows me to combine information across the term structure of inflation expectations in a parsimonious way.

First proposed by Stock and Watson (2007), the unobserved components model is based on the Beveridge and Nelson (1981) decomposition of inflation into permanent and transitory components. The model has been extended variously by Cogley *et al.* (2010) and Cogley and Sargent (2015) and applied to inflation expectations by Kozicki and Tinsley (2012), Crump *et al.* (2018), Nason and Smith (2020), Mertens (2016), Chan *et al.* (2018), and Mertens and Nason (2018).

When applied to expected inflation, the unobserved components model can be used to represent the term structure of inflation expectations at any given point in time as a function of dynamic latent factors, the parameters determining the dynamics of these factors, and the forecast horizon. The model first decomposes actual inflation into a slowly evolving long-run trend component,  $\tau_t$ ; short-run deviations from that trend,  $\tilde{\pi}_t$ ; and a transitory component,  $v_{1,t}$ :

$$\pi_t = \tau_t + \tilde{\pi}_t + v_{1,t}.\tag{1}$$

The two persistent components,  $\tau_t$  and  $\tilde{\pi}_t$  evolve as:

$$\tau_t = \tau_{t-1} + u_{\tau,t},\tag{2}$$

$$\tilde{\pi}_t = a\tilde{\pi}_{t-1} + b\eta'_{t-1} + u_{\tilde{\pi},t},\tag{3}$$

where  $\eta_t$  is a vector of observable macroeconomic variables to be described in greater detail below. Under this specification, inflation can be decomposed into a permanent component,  $\tau_t$ , a temporary but persistent component,  $\tilde{\pi}_t$ , and a purely transitory component,  $v_{1,t}$ . Neither  $\tau_t$  nor  $\tilde{\pi}_t$  depend on lags of the other component but the error terms  $u_{\tau,t}$  and  $u_{\tilde{\pi},t}$ may be correlated. I follow Mertens and Nason (2018) and include a transitory component for inflation in equation (1) so that  $\tau_t$  and  $\tilde{\pi}_t$  can be interpreted as components common to both actual and expected inflation. As I show below, once the dynamics of these variables are known, the unobserved components model can be matched with inflation forecasts at any horizon.

The restrictions that  $\tau_t$  follow a random walk and neither  $\tau_t$  nor  $\tilde{\pi}_t$  have an intercept

term follow from the interpretation of  $\tau_t$  as long-run inflation expectations. Specifically, I interpret this component as:

$$E_t \pi_\infty = \tau_t,\tag{4}$$

which implies that changes in  $\tau_t$  are not forecastable. Were this not the case then forecasters would simply incorporate these adjustments into their current expectations of long-run inflation and hence  $\tau_t$  would not be consistent with long-run expected inflation. For the same reason neither  $\tau_t$  nor  $\tilde{\pi}_t$  have an intercept term. Suppose not and equations (2) and (3) included intercept terms  $\mu_{\tau}$  and  $\mu_{\tilde{\pi}}$ . This would imply that long-run expectations are:

$$E_t \pi_\infty = \tau_t + \mu_\tau + \mu_{\tilde{\pi}},\tag{5}$$

which is inconsistent with equation (4) and the interpretation of  $\tau_t$  as long-run inflation expectations unless  $\mu_{\tau} = \mu_{\tilde{\pi}} = 0$ .

While the short-run component of expectations,  $\tilde{\pi}_t$ , does not depend on lags of  $\tau_t$ , it may depend on its own lag as well as lags of a set of control variables,  $\eta_t$ . Specifically,  $\eta_t$ includes  $y_t$ , the real-time vintage of the growth rate of real Gross Domestic Product (GDP);  $F_t y_{t+4}$ , forecasts of real GDP growth four quarters ahead from the Survey of Professional Forecasters; and  $i_t$ , the interest rate on one-year Treasury bills. The inclusion of forecasts of real GDP growth allows for a forward-looking component of real activity, which should help to identify both variation in the term structure of inflation expectations as well as the central bank's monetary policy rule. I use the last available vintage of real GDP growth before the scheduled due date for the Survey of Professional Forecasters and calculate the interest rate as the daily average between due dates.

Neither real GDP growth nor forecasts of real GDP growth depend on long-run inflation, which can be thought of as imposing the condition that money is super-neutral in the long run, at least in the context of the relatively low-inflation environment over the sample period of 1992–2018. However, because these variables evolve jointly with  $\tilde{\pi}_t$ , I impose no restrictions on the response of real activity variables to short-run inflation dynamics.

This long-run neutrality restriction would not be sensible if applied to the nominal interest rate. It would be hard to imagine, for example, the Federal Reserve allowing an increase in long-run inflation expectations to go unchecked. Furthermore, the linearized Fisher equation states that the nominal interest rate is equal to the sum of the real interest rate,  $r_t$ , and expected inflation over the period of the bond:

$$i_t = r_t + \frac{1}{4} \sum_{j=1}^4 E_t \pi_{t+j}.$$
 (6)

According to the unobserved components model, expected inflation is the sum of expected future values of  $\tau_t$  and  $\tilde{\pi}_t$  so that the nominal interest rate can equivalently be decomposed:

$$i_t = r_t + \tau_t + \frac{1}{4} \sum_{j=1}^4 E_t \tilde{\pi}_{t+j},$$
(7)

which implies that the nominal interest rate and long-run inflation expectations are cointegrated. In order to ensure stability of the system given the unit root in  $\tau_t$ , I impose this cointegrating relationship between the interest rate and long-run inflation expectations and include instead the latent stationary variable  $\tilde{i}_t = i_t - \tau_t$  in the vector of state variables. I additionally de-mean these four state variables so that the vector of macroeconomic variables  $\eta_t$  can be written as:

$$\eta_t = \begin{bmatrix} y_t - \mu_y \\ F_t y_{t+4} - \mu_{Fy} \\ \tilde{i}_t - \mu_i \end{bmatrix}.$$
(8)

Now define the vector of state variables as  $\alpha_t = [\tau_t, \tilde{\pi}_t, \eta_t]$  with dynamics given by the

vector autoregressive process:

$$\alpha_t = B\alpha_{t-1} + u_t,\tag{9}$$

and with restrictions on B as described above to ensure stationarity of the system. With the dynamics of the system known, forecasts of future inflation rates can be constructed by iterating on equation (9). Let  $\iota = [1 \ 1 \ 0 \ ... \ 0]$  be a selection vector which selects and sums the first two elements of any conformable vector. Expected inflation at horizon s can be constructed as:

$$E_t \pi_{t+s} = E_t \tau_{t+s} + E_t \tilde{\pi}_{t+s},\tag{10}$$

$$=\iota B^s \alpha_t. \tag{11}$$

Since optimal forecasts of inflation at any horizon s can be constructed from the dynamics of  $\alpha_t$ , this will be used to link the forecasting data across horizons and uncover the latent factors,  $\tau_t$  and  $\tilde{\pi}_t$ , from the common information in the panel structure of the forecasts and actual inflation. Given B, forecasts of inflation between any two horizons can be included as a measurement equation in a state space model without any additional parameters needing to be estimated. This not only offers considerable dimensionality reduction but also allows for analysis of the entire term structure of expected inflation.

The first measurement equation links the unobserved components model with actual inflation. This was already presented as equation (1) but I repeat it here for completeness and introduce the notation x to generally label measurement equations:

$$x_{1,t} \equiv \pi_t = \tau_t + \tilde{\pi}_t + v_{1,t}.$$
(12)

The remaining measurement equations making up the state space model relate inflation forecasts with the unobserved components model. Before I write these out explicitly I first introduce the forecasting data itself.

#### 3.1 Forecast data

The inflation concept in this paper is seasonally-adjusted Consumer Price Index (CPI) inflation. Although Personal Consumption Expenditure (PCE) inflation is the Federal Reserve's preferred inflation concept, survey measures of expected PCE inflation are available only since 2007. However, the correlation between the two series is quite high at 0.92. Forecasts of CPI inflation are available at a quarterly frequency from the *Survey of Professional Forecasters* and I use the mean forecast as the measure of inflation expectations over the period 1992–2018. Forecasts at a fixed forecast horizon are available for the current and next four quarters ahead. Forecasts at further horizons are also available but they forecast a different inflation concept—average inflation over a given year or period of years, rather than the quarterly inflation rate—and fix the forecast event rather than the forecast horizon. The state space model can accommodate both types of forecasts but the fixed-event forecasts will require special treatment, as I show in detail below.

The next step is to link the unobserved components model with the survey forecasts, which will make up most of the measurement equations in the state space model. Before proceeding directly to this step I first show how to account for information frictions in the data.

#### **3.2** Incorporating information frictions

The unobserved components model provides a framework to describe model-consistent forecasts of expected inflation. One issue with applying the model directly to the survey data, however, is that survey forecasts do not correspond with full-information rational expectations. In particular, forecasters are slow to adjust to new information because of information frictions, so that outdated information plays a significant role in expectations. Coibion and Gorodnichenko (2012) show that when expectations are slow to adjust to new information expected inflation will be less responsive than actual inflation to monetary policy shocks. Mankiw and Reis (2002) propose a model where information is sticky so that only a fraction of agents receive new information in a given period. Those who receive new information use it to make forecasts while those who do not receive new information continue to forecast with out-of-date information.

I now demonstrate how the unobserved components model can be fit to the survey forecasts while accounting for these information frictions. First, let  $F_t \pi_{t+s_1 \to t+s_2}$  denote forecasts of average inflation between periods  $t + s_1$  and  $t + s_2$ . The law of motion for forecasts of average inflation between t and t + s under sticky information is then:

$$F_t \pi_{t \to t+s} = (1 - \omega) E_t \pi_{t \to t+s} + \omega F_{t-1} \pi_{t \to t+s}, \tag{13}$$

where  $\omega$  is the weight placed on outdated information. Forecasts are comprised of two components under this specification: the full-information component,  $E_{t+h}\pi_{t+h+s}$  and the outdated information component,  $F_{t+h-1}\pi_{t+h+s}$ . Notice that, when  $\omega = 0$  the effects of outdated information drop out and the forecasts are model consistent.

Now, if the unobserved components model is a good characterization of full-information inflation expectations, then model predictions at horizon s can be used to replace  $E_t \pi_{t \to t+s}$ :

$$F_t \pi_{t \to t+s} = (1 - \omega) \frac{1}{s} \sum_{j=1}^s \iota B^j \alpha_t + \omega F_{t-1} \pi_{t \to t+s}.$$
 (14)

In the next subsection I show in detail how each of the forecasts I observe in the survey data can be mapped to the unobserved components model when information frictions are present using equation (14).

#### 3.3 Fixed-horizon and fixed-event forecasts

One challenge when working with forecasts from the *Survey of Professional Forecasters* is how to combine information across the term structure given the discrepancy between shortand long-run forecasts. Specifically, while short-run forecasts correspond with a fixed forecast horizon—one-, two-, three-, or four-quarters ahead—longer-run forecasts are fixed-event forecasts where the forecasting period is a fixed window, typically one or more calendar years. For example, forecasts for average inflation in the next year refer to the next calendar year, not necessarily average inflation over the next four quarters. As a result, the forecast window rotates only once per year so that the forecast horizon depends on the quarter in which the forecast is made.

Aruoba (2019) shows how the state space framework can be used to circumvent this problem by splitting up each of the long-run forecasts into four variables, each of which is observed only once per year but now with a common forecast horizon. This essentially exchanges the rotating forecast horizon problem for a missing data problem, which can be handled using the Kalman filter. In this section I show in detail how survey forecasts of both types can be made consistent with the unobserved components model to form a set of measurement equations in the state space model.

I begin with the fixed horizon forecasts, denoted CPI-1, CPI-2, CPI-3, CPI-4, CPI-5, and CPI-6, which correspond with the one-quarter backcast, the nowcast, and the one-, two-, three-, and four-quarter ahead forecasts of the annualized quarterly inflation rate. I follow the naming convention of the actual survey data here. Since the unobserved components model corresponds with the underlying full-information rational expectations, correcting for the presence of outdated information in the surveys requires two forecasts of inflation over the same period taken at different dates, as in equation (13). I do not use the backcast or nowcast because some of actual inflation would be observed when the forecast is made but the nowcast will be useful later. The first of the quarterly forecasts is expected inflation in the next quarter which, recalling the law of motion for inflation expectations is:

$$F_t \pi_{t+1} = (1 - \omega) E_t \pi_{t+1} + \omega F_{t-1} \pi_{t+1}.$$
(15)

As in the previous subsection, let  $\iota = [1 \ 1 \ 0 \ ... \ 0]$  be a selection matrix which selects and sums forecasts of the two unobserved components of expected inflation. One-quarter-ahead inflation expectations from the unobserved components model are then  $\iota B\alpha_t$  and long-run expected inflation is by definition  $\tau_t$ . Relating this to the surveys gives:

$$x_{2,t} \equiv \text{CPI-}3_t = (1-\omega)\iota B\alpha_t + \omega \text{CPI-}4_{t-1} + v_{2,t},\tag{16}$$

which defines the survey as a function of state variables, model parameters, predetermined variables, and measurement error. Equation (16) is the first measurement equation for the survey forecasts. Proceeding in the same manner for the remaining forecasts yields a set of measurement equations which, combined with the vector autoregression for the state variables, will form the state space model.

The next survey, CPI-4, is the forecast of inflation two quarters into the future which, after substituting forecasts from the unobserved components model for the expectations, gives:

$$x_{3,t} \equiv \text{CPI-}4_t = (1-\omega)\iota B^2 \alpha_t + \omega \text{CPI-}5_{t-1} + v_{3,t}.$$
 (17)

The same logic implies:

$$x_{4,t} \equiv \text{CPI-5}_t = (1-\omega)\iota B^3 \alpha_t + \omega \text{CPI-6}_{t-1} + v_{4,t}, \tag{18}$$

which maps the four quarterly forecasts to points along the expected inflation curve.

More distant forecasts from the *Survey of Professional Forecasters* do not correspond with inflation at a single quarter, but rather the average annualized quarterly inflation rate over a fixed forecast window. The first set of these are forecasts of annual inflation in the current year, next year, and year after next, denoted as CPI-A, CPI-B, and CPI-C. I emphasize that these forecasts are with respect to calendar years, not necessarily the next year starting in quarter t + 1. For example, both CPI-B<sub>2004:4</sub> and CPI-B<sub>2004:3</sub> are forecasts of the average annualized quarterly inflation rate in 2005, although the forecasts are made in two different periods and, importantly, have different forecast horizons. As a result, the forecasting concept of the annual forecasts is distinct depending on the quarter in which the forecast is made. Consistent with the notation above, forecasts of annual inflation over the next calendar year made in the fourth quarter are  $F_t \pi_{t\to t+4}$  whereas forecasts of annual inflation over the next calendar year made in the third quarter are  $F_t \pi_{t+1\to t+5}$ .

Because of this feature of the data, I follow Aruoba (2019) and split these variables up so that the forecasts made in each quarter are treated as different variables, each of which is observed only once per year. Denote these CPI-A-Q1, CPI-A-Q2, CPI-A-Q3, CPI-A-Q4, and similarly for CPI-B and CPI-C. Mapping these to the unobserved components model as before gives three measurement equations for the next-year forecasts:

$$x_{5,t} \equiv \text{CPI-B-Q4}_t = (1-\omega) \frac{1}{4} \sum_{i=1}^4 \iota B^i \alpha_t + \omega \text{CPI-B-Q3}_{t-1} + v_{5,t},$$
(19)

$$x_{6,t} \equiv \text{CPI-B-Q3}_t = (1-\omega) \frac{1}{4} \sum_{i=2}^5 \iota B^i \alpha_t + \omega \text{CPI-B-Q2}_{t-1} + v_{6,t},$$
(20)

$$x_{7,t} \equiv \text{CPI-B-Q2}_t = (1-\omega)\frac{1}{4}\sum_{i=3}^6 \iota B^i \alpha_t + \omega \text{CPI-B-Q1}_{t-1} + v_{7,t}.$$
 (21)

In the same way, the year-after-next forecasts map to the unobserved components model with the following three measurement equations:

$$x_{8,t} \equiv \text{CPI-C-Q4}_t = (1-\omega) \frac{1}{4} \sum_{j=5}^8 \iota B^j \alpha_t + \omega \text{CPI-C-Q3}_{t-1} + v_{8,t},$$
(22)

$$x_{9,t} \equiv \text{CPI-C-Q3}_t = (1-\omega) \frac{1}{4} \sum_{j=6}^{9} \iota B^j \alpha_t + \omega \text{CPI-C-Q2}_{t-1} + v_{9,t},$$
(23)

$$x_{10,t} \equiv \text{CPI-C-Q2}_t = (1-\omega) \frac{1}{4} \sum_{j=7}^{10} \iota B^j \alpha_t + \omega \text{CPI-C-Q1}_{t-1} + v_{10,t}.$$
 (24)

The next-year and year-after-next forecasts can also be combined to yield an additional

measurement equation:

$$CPI-B-Q1_t = (1-\omega)E_t\pi_{t+3\to t+7} + \omega CPI-C-Q4_{t-1},$$
(25)

$$x_{11,t} \equiv \text{CPI-B-Q1}_t = (1-\omega)\frac{1}{4}\sum_{j=4}^7 \iota B^j \alpha_t + \omega \text{CPI-C-Q4}_{t-1} + v_{11,t}.$$
 (26)

And finally, the first quarter current-year forecasts can be combined with the next-year forecasts. In this case, however, part of the forecast relates to inflation in the current period:

CPI-A-Q1<sub>t</sub> = 
$$(1 - \omega) \frac{1}{4} (E_t \pi_{t-1 \to t} + 3E_t \pi_{t \to t+3}) + \omega \text{CPI-B-Q4}_{t-1},$$
 (27)

where the first full-information expectation is of inflation in the current period. Since the *Survey of Professional Forecasters* is due in the middle of the quarter the actual inflation rate in that quarter will be partially observed—inflation in the first month of the quarter would be known. Hence this is not a pure forecast. To account for this I make use of the nowcast which, in the presence of information frictions takes the following form:

$$F_t \pi_t = (1 - \omega) E_t \pi_{t-1 \to t} + \omega F_{t-1} \pi_t.$$
 (28)

Rearrange this expression, we can substitute out  $(1 - \omega)E_t\pi_{t-1\to t}$  in equation (27) for observations from the survey data:

$$x_{12,t} \equiv \text{CPI-A-Q1}_{t} = \frac{3}{4}(1-\omega)\sum_{j=1}^{3} \iota B^{j}\alpha_{t} + \omega \text{CPI-B-Q4}_{t-1} + \frac{1}{4}(\text{CPI-2}_{t} - \omega \text{CPI-3}_{t-1}) + v_{12,t}.$$
(29)

Long term five- and ten-year-ahead inflation forecasts are also calendar based, with the reference point being the fourth quarter of the previous year. Hence, the five-year-ahead forecasts made in the first quarter will include the nowcast of the current inflation rate, the second quarter forecasts will include the nowcast and backcast of the previous quarter's inflation rate, and so on. An implication of this feature is that at the time the forecasts are made in the second, third, and fourth quarters, some part of the forecasting object is observable. To account for this I substitute real-time CPI inflation and the nowcast of inflation from the *Survey of Professional Forecasters* for these parts of the forecasts, in the same manner as the previous measurement equation for expected inflation in the current year.

Let CPI-5-Q1 and CPI-10-Q1 denote the five- and ten-year-ahead inflation forecasts made in the first quarter, with the names of forecasts made in the remaining quarters following the same structure. Then:

$$x_{13,t} \equiv \text{CPI-5-Q2}_{t} = (1-\omega)\frac{1}{20} \left(\pi_{t-1} + \sum_{j=1}^{18} \iota B^{j} \alpha_{t}\right) + \omega \text{CPI-5-Q1}_{t-1} + \frac{1}{20}(\text{CPI-2}_{t} - \omega \text{CPI-3}_{t-1}) + v_{13,t},$$

$$x_{14,t} \equiv \text{CPI-5-Q3}_{t} = (1-\omega)\frac{1}{20} \left(\pi_{t-2} + \pi_{t-1} + \sum_{j=1}^{17} \iota B^{j} \alpha_{t}\right) + \omega \text{CPI-5-Q2}_{t-1}$$
(30)

$$+ \frac{1}{20} \left( \operatorname{CPI-2}_{t} - \omega \operatorname{CPI-3}_{t-1} \right) + v_{14,t},$$
(31)

$$x_{15,t} \equiv \text{CPI-5-Q4}_{t} = (1-\omega)\frac{1}{20} \left( \pi_{t-3} + \pi_{t-2} + \pi_{t-1} + \sum_{j=1}^{16} \iota B^{j} \alpha_{t} \right) + \omega \text{CPI-5-Q3}_{t-1} + \frac{1}{20} (\text{CPI-2}_{t} - \omega \text{CPI-3}_{t-1}) + v_{15,t},$$
(32)

$$x_{16,t} \equiv \text{CPI-10-Q2}_{t} = (1-\omega) \frac{1}{40} \left( \pi_{t-1} + \sum_{j=1}^{38} \iota B^{j} \alpha_{t} \right) + \omega \text{CPI-10-Q1}_{t-1} + \frac{1}{40} (\text{CPI-2}_{t} - \omega \text{CPI-3}_{t-1}) + v_{16,t},$$
(33)

$$x_{17,t} \equiv \text{CPI-10-Q3}_{t} = (1-\omega)\frac{1}{40} \left(\pi_{t-2} + \pi_{t-1} + \sum_{j=1}^{37} \iota B^{j} \alpha_{t}\right) + \omega \text{CPI-10-Q2}_{t-1} + \frac{1}{40} (\text{CPI-2}_{t} - \omega \text{CPI-3}_{t-1}) + v_{17,t},$$
(34)

$$x_{18,t} \equiv \text{CPI-10-Q4}_{t} = (1-\omega)\frac{1}{40} \left(\pi_{t-3} + \pi_{t-2} + \pi_{t-1} + \sum_{j=1}^{36} \iota B^{j} \alpha_{t}\right) + \omega \text{CPI-10-Q3}_{t-1} + \frac{1}{40} (\text{CPI-2}_{t} - \omega \text{CPI-3}_{t-1}) + v_{18,t}.$$
(35)

#### 3.4 State space model

The remaining measurement equations correspond with the observable macroeconomic data included in the state vector  $\alpha_t$ : the real-time growth rate of real Gross Domestic Product, forecasts of Gross Domestic Product four quarters ahead, and the interest rate on one-year treasury bills. These enter as deviations from their long-run means, taking the form:

$$x_{19,t} \equiv y_t = \mu_y + \widetilde{y}_t,\tag{36}$$

$$x_{20,t} \equiv F_t y_{t+4} = \mu_{Fy} + \widetilde{F_t y_{t+4}}, \tag{37}$$

$$x_{21,t} \equiv i_t = \mu_i + \tau_t + \tilde{i}_t, \tag{38}$$

where ~ denotes a de-meaned variable.

Let  $x_t = [x_{1,t}, ..., x_{21,t}]'$  be the *N*-dimensional vector of observables outlined above and  $w_t$  a vector of predetermined variables consisting of lags of real-time inflation and survey forecasts. The state space model is:

$$x_t = \mu + \Lambda(B,\omega)\alpha_t + \Phi(\omega)w_t + v_t, \quad v_t \sim N(0,\Sigma_v)$$
(39)

$$\alpha_t = B\alpha_{t-1} + u_t, \qquad u_t \sim N(0, \Sigma_u), \tag{40}$$

where (39) are the measurement equations, derived in the previous subsection, and (40) governs the dynamics of the state variables. The structure of  $\Lambda(B,\omega)$  is determined entirely by B,  $\omega$ , and the forecast horizon of expected inflation in the corresponding row of  $x_t$ . The structure of  $\Phi(\omega)$  is determined entirely by  $\omega$  since lagged forecasts will only enter the law of motion for expectations when information is sticky. Joint estimation of the unobserved components and model parameters is feasible by making use of the model's state space representation. In the next section I describe this estimation procedure.

#### 3.5 Estimation procedure

With the measurement equations specified above I can write the unobserved components model in its state space representation, given by equations (39) and (40), and estimate by maximum likelihood. I follow common practice and assume that the errors in the measurement equations are independent across equations. To reduce the number of parameters to estimate, I also assume the errors in the measurement equations of the inflation forecasts all have the same variance,  $\sigma_F^2$ . Hence  $\Sigma_v$  is a diagonal matrix,  $\Sigma_v = diag(\sigma_{\pi}^2, \sigma_F^2, ..., \sigma_F^2, 0, 0, 0)$ . The state variables follow a VAR(1) process and the covariance matrix  $\Sigma_u$  is unrestricted, allowing for correlation in the error terms across the state variables.

The state space model is non-linear in the parameters, arising from both the information frictions as well as the consistency restrictions. However, for a given set of parameters, the model is linear in the state variables so that the likelihood function can be evaluated with the Kalman filter via the prediction error decomposition. As demonstrated by Aruoba *et al.* (2009), the Kalman filter easily deals with the many missing observations, which arise in the measurement equations of the calendar-year forecasts in the *Survey of Professional Forecasters*, described above. If all variables were observed in a given period then the Kalman filter could proceed as follows. Let  $\Omega_t$  denote the information set at period t and  $a_{t|t} = E[\alpha_t | \Omega_t]$ ,  $a_{t|t-1} = E[\alpha_t | \Omega_{t-1}]$ ,  $P_{t|t} = Var(\alpha_t | \Omega_t)$ , and  $P_{t|t-1} = Var(\alpha_t | \Omega_{t-1})$ . Then,

$$a_{t|t} = a_{t|t-1} + P_{t|t-1}\Lambda'\Sigma_t^{-1}v_t,$$
(41)

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}\Lambda'\Sigma_t^{-1}\Lambda P'_{t|t-1},$$
(42)

$$\Sigma_t = \Lambda P_{t|t-1}\Lambda' + \Phi \Sigma_w \Phi' + \Sigma_v, \tag{43}$$

$$v_t = y_t - \mu - \Lambda a_{t|t-1} - \Phi w_t, \tag{44}$$

$$a_{t+1|t} = Ba_{t|t},\tag{45}$$

$$P_{t+1|t} = BP_{t|t}B' + \Sigma_u,\tag{46}$$

where  $\Sigma_w$  is the covariance matrix for the variables in  $w_t$ .

When some observations are missing in a particular period, define  $N_t^* < N$  as the number of observations that are not missing in period t and  $W_t$  an  $N_t^* \times N$  selection matrix with rows equal to the corresponding row of the identity matrix  $I_N$  if the element of  $x_t$  is observed. Hence, the rank of  $W_t$  is  $N_t^*$ , the number of observed elements of  $x_t$  in period t. Also define,

$$\Lambda^* = W_t \Lambda, \tag{47}$$

$$v_t^* = W_t v_t, \tag{48}$$

$$\Sigma_v^* = W_t \Sigma_v W_t',\tag{49}$$

$$x_t^* = W_t x_t, \tag{50}$$

and the Kalman filter can proceed as before, with these starred variables and parameter matrices in place of their counterparts. The forecast errors from the Kalman filter can then be used to evaluate the log-likelihood function via the prediction error decomposition:

$$\mathcal{L}_{t} = -\frac{1}{2} \left( N_{t}^{*} \log 2\pi + \log |\Sigma_{t}^{*}| + v_{t}^{*'} \Sigma_{t}^{*-1} v_{t}^{*} \right).$$
(51)

The Kalman filter uses information up to period t to evaluate the likelihood and produce estimates of the latent state variables. When we are interested in the latent unobserved components themselves, however, I use the Kalman smoother as described by Durbin and Koopman (2001) to produce estimates of the factors based on all sample data. Specifically, let:

$$K_t = BP_{t|t-1}\Lambda'\Sigma_t^{-1},\tag{52}$$

$$R_t = B - K_t \Lambda, \tag{53}$$

$$d_{t-1} = \Lambda \Sigma_t^{-1} v_t + R_t' d_t, \tag{54}$$

$$M_{t-1} = \Lambda' \Sigma_t^{-1} \Lambda + R_t' M_t R_t, \tag{55}$$

$$a_{t|T} = a_{t|t-1} + P_{t|t-1}d_{t-1}, (56)$$

$$P_{t|T} = P_{t|t-1} - P_{t|t-1} M_{t-1} P_{t|t-1}, (57)$$

and initial values  $d_T = 0$  and  $M_T = 0$ . Iterating on these equations backwards from T - 1 gives optimal estimates of the state variables based on all sample data, which I will use when analyzing the unobserved components of expected inflation themselves. Note that because the likelihood function is based on the forecasts produced by the Kalman filter, use of the Kalman smoother has no impact on the estimated model parameters.

I maximize the log-likelihood function, the sum over all t of equation (51), using nonlinear optimization methods and numerical derivatives, which I compute as the change in the likelihood induced by a small change in parameter values. Specifically, let  $\theta$  denote the k-dimensional column vector of model parameters. The score is then a  $k \times T$  matrix calculated via two-sided numerical derivatives such that the row *i* column *t* element is:

$$S_{i,t} = \frac{\mathcal{L}_t(\theta + q) - \mathcal{L}_t(\theta - q)}{2q_i}, \quad \text{where } q_j = \begin{cases} \Xi, & \text{for } i = j \\ 0, & \text{otherwise} \end{cases}$$
(58)

and  $\Xi$  is the square root of machine precision. For a given parameter vector,  $\theta^{(l)}$ , I calculate the parameter vector at the  $(l+1)^{th}$  iteration with Newton's method:

$$\theta^{(l+1)} = \theta^{(l)} - \kappa^{(l)} H^{(l)} f^{(l)}, \tag{59}$$

where  $\kappa^{(l)}$  is the step size,  $H^{(l)}$  is an approximation of the Hessian and  $f^{(l)}$  is the negative of the gradient vector, which has typical element  $f_i^{(l)} = -\sum_{t=1}^T \mathcal{S}_{i,t}^{(l)}$ . I use the Broyden-Fletcher-Goldfarb-Shanno (BFGS) procedure to approximate the Hessian matrix of second derivatives. Let  $\Delta \theta^{(l+1)} = \theta^{(l+1)} - \theta^{(l)}$  and  $\Delta f^{(l+1)} = f^{(l+1)} - f^{(l)}$ . Then the approximate Hessian at the next iteration is updated as:

$$H^{(l+1)} = H^{(l)} - \frac{H^{(l)} \Delta \theta^{(l+1)} \Delta \theta^{(l+1)'} H^{(l)}}{\Delta \theta^{(l+1)'} H^{(l)} \Delta \theta^{(l+1)}} + \frac{\Delta f^{(l+1)} \Delta f^{(l+1)'}}{\Delta f^{(l+1)'} \Delta \theta^{(l+1)'}}.$$
(60)

I initialize by setting  $H^{(0)} = 10^6 I_k$ , so that equation (59) begins with a steepest descent step. As the number of iterations increases, more curvature information is incorporated into the approximate Hessian which speeds up convergence.

The BFGS updating formula is valid only when the condition  $\Delta \theta^{(l+1)'} \Delta f^{(l+1)} > 0$  holds. When this condition is violated—which occurs only rarely—I instead update with a damped BFGS formula proposed by Powell (1978) which takes as the change in the gradient:

$$\Delta f^{(l+1)*} = \delta \Delta f^{(l+1)} + (1-\delta) H^{(l)} \Delta \theta^{(l+1)}, \tag{61}$$

$$\delta = 0.8 \frac{\Delta \theta^{(l+1)'} H^{(l)} \Delta \theta^{(l+1)}}{\Delta \theta^{(l+1)'} H^{(l)} \Delta \theta^{(l+1)} - \Delta \theta^{(l+1)'} \Delta f^{(l+1)}},$$
(62)

with  $\Delta f^{(l+1)*}$  taking the place of  $\Delta f^{(l+1)}$  in equation (60).

To choose the step size,  $\kappa^{(l)}$ , I begin with an initial value of unity and then proceed with a line search by bisection, taking the first candidate to satisfy the Wolfe conditions using the algorithm proposed by Nocedal and Wright (2006). In order to ensure that the covariance matrices are always positive semi-definite, I reparameterize them by first taking their Cholesky decomposition and then setting the main diagonal equal to its natural logarithm before stacking in  $\theta$ . Derivatives are then taken with respect to these new reparameterized variables. Of course, when evaluating the log-likelihood function this procedure is done in reverse so that there is no effect on the log-likelihood function itself, but this ensures that the optimization algorithm will never take a step resulting in a negative variance estimate.

## 4 Results

#### 4.1 Estimation results and model evaluation

The model provides evidence of information frictions: I estimate  $\hat{\omega} = 0.2823$  with a standard error, calculated as the diagonal element of the inverse Hessian matrix, of 0.0270. Under the interpretation of the sticky information model, this indicates that approximately 28%of professional forecasters are operating with outdated information sets and that forecasters update their information sets every  $\frac{1}{1-\hat{\omega}} = 1.39$  quarters, on average. This is very close to the findings of Mertens and Nason (2018) who estimate a value of approximately 0.3 for GDP deflator inflation, despite differences in sample period, inflation concept, as well as forecast horizon. It also matches closely the estimates of the sticky information parameter in forecasts of CPI inflation reported by Coibion and Gorodnichenko (2015), who propose a new regression approach to test for the presence of information rigidities. On the other hand, it is somewhat smaller than the estimate of 0.438 reported by Nason and Smith (2020), which may be explained by their inclusion of the 1980s in their sample, which they are able to do because they focus only on short-run expectations. Interestingly,  $\omega$  is larger than most of the estimates reported by Döpke et al. (2008) and Andrade and Le Bihan (2013) who provide estimates of the Sticky Information parameter for European countries using the European Survey of Professional Forecasters.

To further demonstrate statistical evidence supporting the presence of information frictions in the survey forecasts, I calculate a likelihood ratio test, comparing the model with and without sticky information. The test statistic is:

$$2\left(\sum_{t=1}^{T} \mathcal{L}(\hat{\theta}, \hat{\alpha}_t) - \sum_{t=1}^{T} \mathcal{L}(\bar{\theta}, \bar{\alpha}_t)\right) \sim \chi^2(1), \tag{63}$$

where  $\sum_{t=1}^{T} \mathcal{L}_t(\hat{\theta}, \hat{\alpha}_t)$  is the likelihood function evaluated at the estimated model parameters  $\hat{\theta}$  and state variables  $\hat{\alpha}_t$ , and  $\bar{\theta}$  and  $\bar{\alpha}_t$  denote the restricted estimates. The tests statistic is 67.75 with a p-value of essentially zero, indicating strong evidence of information frictions in the survey data.

Table 1 shows parameter estimates for the matrix B in equations (40), which determines the dynamics of the state variables. Standard errors, in parentheses, are calculated as the diagonal elements of the inverse Hessian matrix. The elements of B associated with  $\tau_t$  are excluded since  $\tau_t$  neither depends upon nor influences the other variables. Both actual and forecasts of GDP enter significantly in equations for the interest rate and short-run inflation expectations. In each case, the variables are more responsive to a unit increase in forecasts than actual GDP, indicating the importance of including a forward-looking real activity component to explain the dynamics of expected inflation and the central bank's monetary policy rule.

Figure 1 shows estimates of the two unobserved components,  $\tau_t$  and  $\tilde{\pi}_t$ , extracted with the Kalman smoother, for the benchmark model with sticky information (solid lines) as well as the restricted model with no information frictions (dashed lines). The shaded areas are the 68% and 90% confidence intervals associated with the estimated latent factors from the benchmark model, calculated using Monte Carlo methods as proposed by Hamilton (1986).

The benchmark long-run component,  $\tau_t$ , has a mean of 2.84%, somewhat larger than the average inflation rate between 1992 to 2018 (2.25%). The discrepancy can be explained by two effects. First, from 1992 to 2000 long-run inflation expectations were slowly adjusting downward from the higher inflation period in the 1980s. Second, inflation remained below long-run expectations during the recovery from the financial crisis and ensuing recession in 2008. Between these two episodes, from 2000 to 2009, the level factor has a mean of 2.69% comparable with 2.54% for actual inflation. While the most prominent feature of the longrun component is the large downward adjustment over the first eight years of the sample, long-run inflation expectations also demonstrate non-trivial variation over the remaining sample period. This includes two relatively large increases over a short period of time: first in 2000 and again in 2007 just as the Federal Reserve was lowering interest rates in response to the financial crisis.

The short-run component of expected inflation,  $\tilde{\pi}_t$ , is much more variable than the longrun component and is typically below zero, which indicates that short-run inflation expectations were typically below long-run inflation expectations over the sample period. This is especially true around 2008 as the economy fell into recession but also extends through the recovery period until the end of the sample as inflation remained surprisingly low during the economic recovery. Figure 1 indicates that this low inflationary period was interpreted as temporary by forecasters. Hence the short-run component is negative while long-run expectations fluctuate around their long-run mean, although there is some indication that long-run inflation expectations are beginning to adjust downwards to a new lower-inflation environment.

Comparing the solid and dashed lines in Figure 1 gives an indication of when and by how much information frictions matter. In most cases, the restricted estimates fall within the confidence intervals of the estimated unrestricted components. However, there are some periods where the latent factors are further apart than others. The long-run component, for example, shows that sticky information plays a larger role in the second half of the sample. The effect is typically significant at the 68% level but holds for the final ten years of the sample, which appears to suggest that information frictions have not become less important over time.

Figure 2 shows the term structure of inflation expectations from the unobserved components model at two dates, with and without information frictions. In the second quarter of 2005, a period of relative economic calm, expected inflation from the restricted and unrestricted models looks very similar. The economy is relatively stable so that new information does not significantly alter existing forecasts. However, as the economy is entering a severe recession in the fourth quarter of 2008, the two models provide very different estimates of expected inflation over the short run. The benchmark unrestricted model with sticky information shows expected inflation is below average in the short run and a full two percentage points below the model with no frictions.

The figure demonstrates that, when economic conditions are very volatile, old information can be very different from new information so that forecasts based on the two information sets can be quite different. To further demonstrate this point, define:

$$\zeta_{s,t} = |\iota \hat{B}^s \hat{\alpha}_t - \iota \bar{B}^s \bar{\alpha}_t|, \tag{64}$$

as the absolute difference between the predictions of the unrestricted model with sticky information and restricted model without. If this difference is on average large, that indicates that the information frictions matter not just statistically but economically. I estimate the following regressions:

$$\zeta_{s,t} = a_s + b_s \gamma_t + error_{s,t},\tag{65}$$

where  $\gamma_t$  is a dummy variable taking the value of one in a recession, according to the NBER recession dates, and zero otherwise. Then,  $a_s$  has the interpretation as the average difference between expected inflation at horizon s uncovered from the restricted and unrestricted models in normal times and  $a_s + b_s$  the average difference between the two models in recessionary periods.

Figure 3 shows estimates of  $a_s$  and  $b_s$  in equation (65) for s = 0, ..., 40 where  $\zeta_{s,t}$  is the absolute difference between the predictions of the unrestricted model and the restricted model with no information frictions (ie.  $\omega = 0$ ). Solid line segments indicate the estimated coefficient is statistically significant at the 10% level and dashed line segments indicate otherwise. The top panel shows the average effect of information frictions on estimates of inflation expectations by forecast horizon. The largest effect is for the contemporaneous expectations, which differ by approximately 26 basis points, on average. The effect is smaller for the remaining forecast horizons, typically between 5 and 10 basis points, but is statistically

significant at all horizons.

The bottom panel of Figure 3 shows that the difference between the restricted and unrestricted models is larger in recessionary periods. Again the largest effects occur at the short end of the term structure. Taken together, the estimates in Figure 3 indicate that there is a difference of approximately 66 basis points between the models with and without information frictions in recessionary periods for expectations of inflation in the current period, a sizable value given that average inflation over the sample period is 2.25%. As the forecast horizon increases, the extra effect of information frictions in recessionary periods declines quickly, and is not statistically significant in several cases.

#### 4.2 Impulse response functions

Identification of the effects of monetary policy is a challenging problem because monetary policy both influences and actively responds to current and anticipated economic conditions. For this reason contemporaneous adjustments to the nominal interest rate are endogenous with respect the the remaining variables in  $\alpha_t$  and standard OLS estimates of the contemporaneous interest rate effect are not reliable.

Consider, however, a variable  $\epsilon_{t,i}$  which represents the exogenous component of changes to the interest rate, a monetary policy shock. Were  $\epsilon_{t,i}$  observed then the effects of monetary policy shocks on  $\alpha_t$  could be estimated consistently and dynamic effects traced out using the estimate of *B* already obtained. Unfortunately,  $\epsilon_{t,i}$  is not directly observable. However, we do know that these shocks are related to the error terms  $u_t$  in the state equations (40) which, although they have no economic interpretation beyond forecast errors, contain all unpredictable variation in the state equations. Some of this variation will be due to the monetary policy shocks so that we can express the forecast errors as:

$$u_t = \Theta_i \epsilon_{t,i} + v_t, \tag{66}$$

where  $\Theta_i$  gives the contemporaneous effect of the monetary policy shocks on each variable in the system and  $v_t$  is an error term which is a linear combination of the remaining shocks driving the system,  $\epsilon_{t,j}$  for  $j \neq i$ . The impulse response functions of the state variables hquarters after a monetary policy shock can then be written as:

$$\Psi_h = \frac{\partial \alpha_{t+h}}{\partial \epsilon'_{t,i}} = J B^h J' \Theta_i, \tag{67}$$

where  $J = [I_m \ 0_{m \times m} \ \cdots \ 0_{m \times m}]$  is a selection matrix, and *m* the number of state variables.

Although  $\epsilon_{t,i}$  is itself unobservable, Mertens and Ravn (2013) and Stock and Watson (2012) show that the effect of this shock can be identified given a suitable instrument  $z_t$  satisfying the following relevance and exclusion conditions:

$$E[z_t \epsilon_{t,i}] \neq 0, \tag{68}$$

$$E[z_t \epsilon_{t,j}] = 0 \quad \text{for } j \neq i.$$
(69)

If these conditions are satisfied, then  $\Theta_i$  can be estimated consistently from the regression:

$$\hat{u}_t = \Theta_i \hat{u}_{t,i} + \nu_t, \tag{70}$$

by instrumenting for  $\hat{u}_{t,i}$  with  $z_t$ .

Following Gertler and Karadi (2015), I construct an instrument from the change in federal funds futures rates in a 30-minute window around monetary policy announcements, using an updated set of the Federal Open Market Committee meeting dates from Gürkaynak *et al.* (2005b). To get the final instrument, I follow Miranda-Agrippino and Ricco (2017) and use the residuals from a regression of changes in the futures contracts on four of their own lags, and forecasts and forecast revisions from the Federal Reserve's Greenbook. I include the nowcast, backcast, and forecasts of CPI inflation and real GDP in the next three quarters, the nowcast of the unemployment rate and revisions of the nowcast, backcast, and forecasts of inflation, real GDP, and the unemployment rate in the next two quarters. This is to control for information frictions as well as differences in the information sets of policymakers and financial market participants.

Valid inference requires that the instrumental variable is sufficiently correlated with the endogenous variable, typically indicated with a large F-statistic from the first-stage regression. The first-stage regressions have F-statistics of 9.82 and 9.24 for the restricted and unrestricted models, respectively. Stock *et al.* (2002) observe that whether or not an instrument should be considered weak depends on the tolerance for departures from the usual standards of inference. For example, the requirement that a 5% hypothesis test rejects no more than 15% of the time requires a first-stage F-statistic of 8.96. The F-statistics from both first-stage regressions exceed this threshold, so weak instruments do not appear to be of concern.

I calculate asymptotic standard errors for the impulse response functions via the delta method. Let  $\beta = \text{vec}(B)$  and assume that:

$$\sqrt{T} \begin{pmatrix} \hat{\beta} - \beta \\ \hat{\Theta}_i - \Theta_i \end{pmatrix} \xrightarrow{d} N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{\beta} & 0 \\ 0 & \Sigma_{\Theta_i} \end{bmatrix} \right).$$
(71)

The delta method says that for a vector of parameters b satisfying:

$$\sqrt{T}(\hat{b}-b) \xrightarrow{d} N(0,\Sigma_b),$$
(72)

and a continuous differentiable function  $g(\cdot)$ , the transformation of the variables follows the distribution:

$$\sqrt{T}(g(\hat{b}) - g(b)) \xrightarrow{d} N\left(0, \frac{\partial g}{\partial b'} \Sigma_b \frac{\partial g'}{\partial b}\right).$$
(73)

Using this result, the impulse response functions follow the asymptotic distribution:

$$\sqrt{T}(\hat{\Psi}_h - \Psi_h) \xrightarrow{d} N(0, A_h \Sigma_\beta A'_h + \bar{A}_h \Sigma_{\Theta_r} \bar{A}'_h), \tag{74}$$

where,

$$A_{h} = \begin{cases} 0 & \text{for } h = 0, \\ (\Theta_{i}^{\prime} \otimes I_{m}) \left( \sum_{j=0}^{h-1} J(B^{\prime})^{h-1-j} \otimes JB^{j}J^{\prime} \right) & \text{for } h \ge 1, \end{cases}$$

$$(75)$$

and

$$\bar{A}_h = JB^h J' \quad \forall \quad h. \tag{76}$$

This result follows from Lütkepohl (1990) but the calculations differ slightly since I am only interested in the single impulse response vector,  $\Theta_i$ . I show them here for completeness. First, notice that the additive nature of the covariance matrix in equation (74) follows from the block diagonal structure of equation (71). Then, to derive the matrices  $A_h$  and  $\bar{A}_h$  is just a matter of matrix differentiation.

First, notice that when h = 0,  $\Psi_0 = J J' \Theta_i$ , so that:

$$A_0 = \frac{\partial \Psi_0}{\partial \beta'} = 0. \tag{77}$$

For h > 1:

$$\frac{\partial \Psi_h}{\partial \beta'} = \frac{\partial (JB^h J'\Theta_i)}{\partial \beta'},\tag{78}$$

$$=\frac{\partial \operatorname{vec}(JB^{h}J'\Theta_{i})}{\partial\beta'},\tag{79}$$

$$= (\Theta'_i J \otimes J) \frac{\partial \text{vec}(B^h)}{\partial \beta'}, \tag{80}$$

$$= (\Theta'_i \otimes I_m) \left( \sum_{j=0}^{h-1} J(B')^{h-1-j} \otimes JB^j J' \right), \tag{81}$$

where I omit the steps between the final two lines because these are standard and follow

directly from Lütkepohl (1990). Now, for  $\bar{A}_h$ :

$$\frac{\partial \Psi_h}{\partial \Theta'_i} = \frac{\partial (JB^h J'\Theta_i)}{\partial \Theta'_i},\tag{82}$$

$$=JB^{h}J',$$
(83)

which gives the result. I estimate the covariance matrix of autoregressive coefficients,  $\Sigma_{\beta}$  from the inverse of the Hessian matrix evaluated at the solution parameters. I use a sandwich estimator for the covariance matrix  $\Sigma_{\Theta_i}$ . Let  $P_z = z'(zz')^{-1}z$  be the matrix projecting onto the space spanned by z, the instrumental variable. Then, the IV estimator for equation (70) can be written:

$$\hat{\Theta}_i = (\hat{u}_i P_z \hat{u}_i \otimes I_m)^{-1} (\hat{u}_i P_z \otimes I_m) vec(\hat{u}).$$
(84)

The sandwich covariance matrix estimator, which allows for correlation across the error terms in the equations (70), is:

$$\Sigma_{\Theta_i} = (\hat{u}_i P_z \hat{u}_i \otimes I_m)^{-1} (\hat{u}_i P_z \otimes I_m) (I_T \otimes \Sigma_\nu) (P_z \hat{u}_i \otimes I_m) (\hat{u}_i P_z \hat{u}_i \otimes I_m)^{-1}, \tag{85}$$

where  $\hat{\Sigma}_{\nu} = \frac{1}{T}\hat{\nu}\hat{\nu}'$  and  $\hat{\nu} = \hat{u} - \hat{\Theta}_i P_z \hat{u}_i$ .

Figure 4 shows impulse response functions of the short- and long-run components after a monetary policy shock normalized to raise the detrended one-year treasury bill rate by 100 basis points. The maximum impulse response horizon is 40 quarters, which coincides with the longest available inflation forecast in the *Survey of Professional Forecasters*. The solid black lines correspond with the responses of the components from the benchmark model with sticky information. The light and dark shaded areas are the associated 68% and 90% asymptotic confidence intervals calculated as described above. For comparison, the dashed lines show the response of the components from the restricted model with no sticky information.

Long-run inflation expectations decrease significantly upon impact after a contractionary

monetary policy shock and, because the process follows a random walk, the effect is permanent. This is a somewhat surprising result, and runs counter to the conventional view of monetary policy. If the monetary authority has a credible long-run inflation target, then we would expect long-run inflation expectations to be stable at this target and hence not respond to a temporary policy change. Here, long-run inflation expectations decline by 14 basis points. This result is also not sensitive to the exclusion of sticky information.

The short-run component does not respond contemporaneously to a monetary policy shock. In the periods following the policy change, the short-run component first increases and then declines as the policy effect wears off over the course of several years. The effect is temporary because, unlike the long-run component,  $\tilde{\pi}_t$  is stationary. The positive response of the short-run component is somewhat smaller in the model without information frictions, and occasionally the differences are statistically significant, but the overall shape and timing of the responses are similar.

Because of the additive structure of the unobserved components model, the response of expected inflation is simply the sum of the responses of the short- and long-run component, which is shown in the top panel of Figure 5. Over the first several years, the positive response of the short-run component essentially cancels out the negative response of the long-run component, so that overall there is no significant response of inflation expectations. However, because the response of the long-run component is permanent, eventually it will dominate as the effect of the stationary short-run component fades away.

In the benchmark model there are several horizons for which the response of expected inflation is positive—between two and ten quarters after the policy is implemented—but the effect is never significant at the 10% level. In the restricted model, the response of expected inflation is never positive. In the short-run, then, the response of expectations appears consistent with the typical response of actual inflation: there is no significant response for several years, after which there is a significant negative response.

One reason to expect that expectations may not respond in the same manner as actual

inflation is the information channel of monetary policy, whereby policy changes communicate the Federal Reserve's superior information to forecasters so that expectations adjust in the same direction as the policy change in the short-run. For example, forecasters may interpret an unanticipated monetary policy tightening as an indication that their expectations of inflation were too low and hence revise them upwards over the short term. Figure 5 shows that this is not the case, so that monetary policy decisions do not appear to communicate information about current or short-term inflation rates. This is consistent with Nakamura and Steinsson (2018), who find no response of inflation expectations to a monetary policy shock in the short run and a negative response in the long-run, and suggests a strong response of real interest rates to a monetary policy shock.

An implication of the finding that long-run inflation expectations decline permanently after a monetary policy shock is that future interest rates will eventually decline by the same amount. In other words, higher interest rates today cause lower interest rates in the future through an inflation expectations channel. This follows directly from the cointegrating relationship between interest rates and long-run inflation expectations. The timing of this depends on the persistence of the response of the stationary component of the nominal interest rate. The bottom panel of Figure 5 shows the response of the interest rate after a monetary policy shock. Interest rates remain positive for several years after the contractionary monetary policy shock. Eventually the response becomes insignificantly different from zero, but does not turn negative even ten years after the policy change because of the high degree of persistence in the stationary component of nominal interest rates.

#### 4.3 Historical and variance decompositions

Given the underlying monetary policy shocks,  $\epsilon_{t,i}$ , an additional object of interest is the historical contribution of specific exogenous monetary policy changes to fluctuations of the two components of expected inflation. This gives an indication of periods where the influence of monetary policy on expectations was particularly big or small. As demonstrated by Stock and Watson (2018), the monetary policy shocks can be uncovered as:

$$\epsilon_{t,i} = \frac{\Theta_i' \Sigma_u^{-1}}{\Theta_i' \Sigma_u^{-1} \Theta_i} u_t.$$
(86)

Given the monetary policy shocks, the cumulative contribution of the estimated monetary policy shocks to changes in the state variables over the sample period is given by:

$$\Delta \alpha_t = \sum_{h=0}^{\infty} J B^h J' \Theta_i \epsilon_{t-h,i}.$$
(87)

Figure 6 shows the cumulative effect of monetary policy shocks on the two unobserved components over the sample period. Notice that, because the long-run component follows a random walk, the effect of the monetary policy shocks never wears off. As a result, the cumulative effect of monetary policy shocks at any given date is simply the sum of the estimated monetary policy shocks since the start of the sample, scaled by the contemporaneous response of the long-run component to monetary policy shocks, estimated to be -0.14. So, the historical decomposition of the long-run component demonstrates both the cumulative response of long-run inflation expectations to monetary policy shocks over the sample as well as the (scaled) cumulative sum of the monetary policy shocks themselves.

The cumulative change in long-run inflation expectations that can be explained by monetary policy shocks is initially small and positive, but quickly falls below zero as forecasters begin to adjust to the reality of lower inflation, consistent with Figure 1. The historical decomposition indicates that monetary policy actions undertaken at the time can only partially explain this effect. In the four years between the beginning of 1993 and the end of 1996, monetary policy decreased inflation expectations by approximately 40 basis points. Long-run inflation expectations continued to decline through the remainder of the 1990s, but this does not appear to be because of monetary policy actions, which were essentially neutral through that time in the benchmark model.

The remainder of the sample includes two periods of consecutive monetary policy easings

which put upward pressure on long-run expectations. The first begins in 2000 as the Federal Reserve responded to worsening economic conditions after the dot-com stock market crash. These policy actions directly increased long-run inflation expectations by approximately 60 basis points. The second period of easings coincides with the financial crisis beginning in 2007. First, there are three consecutive easings in the final two quarters of 2007 and the first quarter of 2008. Following that, monetary policy was initially contractionary during the first asset purchase program (QE1) which likely reflects that markets anticipated a more aggressive program than what the Federal Reserve ultimately implemented.

In the fourth quarter of 2008 the Federal Reserve set the target range for the federal funds rate to between 0 and 1/4 percent, effectively hitting the zero lower bound, where it remained until December 2015. Hence, monetary policy shocks throughout this period can be attributed to either forward guidance or asset purchase programs undertaken by the Federal Reserve. Forward guidance consisted of communicating information about both future monetary policy decisions—promising to keep interest rates low beyond the next meeting date—as well as specifying economic preconditions that would need to be met until the Federal Reserve would consider lifting the policy rate off its lower bound. Specifically, the Federal Reserve frequently indicated that interest rates would remain at their "exceptionally low level" while employment remained below its maximum level—often explicitly stated as an unemployment rate above 6.5%—medium-term inflation was projected to be no greater than 2.5%, and long-run inflation expectations remained well anchored.

Of the 28 quarters making up the zero lower bound period, 23 have negative monetary policy shocks, indicating that, despite the constraint on short-term interest rates, the Federal Reserve was able to influence longer term interest rates through unconventional monetary policies. The absolute mean value of monetary policy shocks over this period is 22 basis points compared with 28.9 basis points over the full sample, indicating that while there may have been some limitations to the effectiveness of monetary policy when constrained at the zero lower bound, overall these limitations were small. Figure 6 shows that by the end of 2014 the cumulative effect of monetary policy actions over the previous five years contributed to increase long-run expectations by 60 basis points. Again, because short-run policy rates were constrained at the zero lower bound during this period, this effect can be attributed to unconventional monetary policy operations. This finding is consistent with Boneva *et al.* (2016) who find that unconventional monetary policy actions taken by the Bank of England had a significant effect on inflation expectations of firms in the United Kingdom.

Following the termination of the final asset purchase program there has been downward pressure on long-run inflation expectations. Between 2015 and 2017 monetary policy was essentially neutral—monetary policy shocks are all very close to zero during this time—and the contribution of earlier expansionary policy on expectations begins to level out. Finally, in late 2017 and 2018 there are two consecutive contractionary monetary policy shocks as the Federal Reserve looked to normalize interest rates, which appear to have contributed to the trend of lower long-run inflation expectations. But, this cannot completely explain the downward trend in long-run inflation expectations in the final five quarters of the sample; long-run inflation expectations have decline by 20 basis points, only one-quarter of which can be explained by monetary policy.

By contrast, in the restricted model which does not account for sticky information, the contribution of monetary policy to changes in long-run inflation expectations is notably smaller. This is true over the full sample period, over which monetary policy is essentially neutral, but is particularly noticeable over the zero lower bound period. This can be explained by two effects. First, as demonstrated by the impulse response functions, the response of long-run expectations to monetary policy is somewhat smaller in the restricted model. While this difference is small and not statistically significant, the cumulative effect can be large. Second, the two models have different estimated monetary policy shocks. For example, the cumulative sum of monetary policy shocks over the zero lower bound period differ by 108 basis points, with the unrestricted model finding that monetary policy was substantially more accommodating over this period than the restricted model.

The bottom panel of Figure 6 shows the historical decomposition of the short-run component of inflation expectations. Over much of the sample this mirrors the response of long-run expectations, consistent with earlier findings that the responses of these two components to monetary policy offset eachother in the short run. However, because the short-run component is stationary, the effect of previous monetary policy shocks becomes smaller as the shocks become more distant.

Figure 7 shows the forecast error variance decomposition for the unobserved components of the benchmark model, which indicates the fraction of the variance of the forecast errors at horizon H,  $\hat{\alpha}_{t+H} - \hat{B}^H \hat{\alpha}_t$ , that can be explained by monetary policy shocks. For a given variable, I calculate these as:

$$\frac{\Theta_i'\left(\sum_{h=0}^H \left(JB^h J'\right)' e_j e_j' JB^h J'\right) \Theta_i}{\left(\Theta_i' \Sigma_u^{-1} \Theta_i\right) e_j' \left(\sum_{h=0}^H \left(JB^h J'\right)' \Sigma_u JB^h J'\right) e_j},\tag{88}$$

where j is the position of the variable in the vector  $\alpha_t$ , and  $e_j$  is a selection vector with a one in position j and zeroes elsewhere. This gives an indication of how important monetary policy shocks are to explain the dynamics of expected inflation overall.

In the benchmark model, nearly 40% of the variation in long-run expectations can be explained by monetary policy shocks. Once again, because this component follows a random walk, the effect is the same at all horizons. By contrast, much less of the variation in the short-run component can be explained by monetary policy shocks. At short horizons, monetary policy explains virtually none of the variation in the short-run component, and even at longer horizons only 5% of the variation can be attributed to monetary policy. We also see that, consistent with the historical decompositions, the model with information frictions finds that monetary policy explains much more of the variation of inflation expectations than the restricted model.

## 5 Conclusion

This paper estimates the impact of monetary policy on the term structure of inflation expectations. To do this I use an extension of the unobserved components model which accounts for information frictions and includes additional macroeconomic variables. I then use the model to identify the effects of monetary policy on expectations using SVAR-IV methods. I find significant evidence of information frictions. Specifically, 28% of inflation expectations consist of outdated information. I also show that the effects of sticky information on expectations matter more during recessions, especially around the recession beginning in 2008, and at short forecast horizons.

I then show that the long-run component of inflation expectations declines after a monetary policy tightening and that, because this component follows a random walk, the effect is permanent. The short-run component moves in the opposite direction so that, over the first several years there is no response of expected inflation. But, because the short-run component is stationary, the long-run effect eventually dominates and expected inflation falls. This result holds whether or not information frictions are accounted for in the model, although the benchmark model with information frictions does find somewhat larger effects.

I also find that a much larger share of the variation in long-run expectations can be explained by monetary policy actions than for short-run expectations. Specifically, monetary policy put sustained upward pressure on long-run expectations from 2009 to 2014, helping to prop up inflation expectations through the financial crisis and economic recovery. Since then long-run expectations have demonstrated a downward trajectory, some of which can be attributed to increases in the policy rate in 2018, but most of which appears due to other factors.

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## A Tables and Figures

	$\tilde{\pi}_{t-1}$	$\tilde{y}_{t-1}$	$\widetilde{F_{t-1}}y_{t+3}$	$\tilde{i}_{t-1}$
$\tilde{\pi}_t$	0.3765	-0.0585	-0.1073	0.1205
	(0.0435)	(0.0264)	(0.0347)	(0.0188)
$\widetilde{y}_t$	0.1953	0.2882	0.0110	0.0822
	(0.1361)	(0.0706)	(0.0938)	(0.0612)
$\widetilde{F_t y}_{t+4}$	-0.0478	0.0049	0.8085	0.0115
	(0.0326)	(0.0176)	(0.0485)	(0.0125)
$\widetilde{i}_t$	-0.0089	0.0616	-0.1272	0.9439
	(0.0357)	(0.0193)	(0.0492)	(0.0170)

Table 1: Estimates of transition matrix B

Note: Parameter estimates for the matrix B in the state equations (40). Rows correspond with the dependent variable and columns explanatory variables. Standard errors in parentheses are the diagonal elements of the inverse Hessian matrix. Excluded are the first row and first column of Bsince  $\tau_t$  follows a random walk and is restricted so that its lags affect no other state variables.



Figure 1: Estimated unobserved components of the state space model

Note: Estimates of the unobserved long- and short-run components from the state space model. The black solid line shows the estimated unobserved components from the model allowing for sticky information. The dark and light shaded areas are the associated 68% and 90% asymptotic confidence intervals. For comparison, the dashed lines show the unobserved components for models with no sticky information ( $\omega = 0$ ). I estimate the factors with the Kalman smoother and construct the confidence intervals by Monte Carlo simulation as proposed by Hamilton (1986) based on 5000 draws of the model parameters.

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#### Figure 2: Expected inflation at two dates



Note: Expected inflation from the unobserved components model at two dates. The black solid lines show expected inflation from the unobserved components model with sticky information. For comparison, the dashed lines show expected inflation from the models with no sticky information ( $\omega = 0$ ).



Figure 3: Estimates from regression:  $\zeta_{s,t} = a_s + b_s \gamma_t + error_{s,t}$ 

Note: Estimated coefficients from regression (65), where  $\gamma_t$  is and indicator variable equal to one in an NBER recession period and zero otherwise and  $\zeta_{s,t}$  is the absolute difference between the predictions of the unrestricted model and the restricted model with no information frictions (ie.  $\omega = \rho = 0$ ). Solid lines indicate the estimated coefficient is significant at the 10% level and dashed lines otherwise.



Figure 4: Response of unobserved components to a monetary policy shock

Note: Estimated impulse response functions of the short- and long-run components of the unobserved components model after a monetary policy shock normalized to raise the detrended one-year treasury bill rate by 100 basis points. The black solid line shows the response of the unobserved components from the model allowing for sticky information, where  $\omega$  was estimated to be 0.2823. The dark and light shaded areas are the associated 68% and 90% asymptotic confidence intervals. For comparison, the dashed line shows the response of the unobserved components from the model with no information frictions ( $\omega = 0$ ). Detailed calculations of the impulse response functions and confidence intervals are outlined in Section 4.2.





Note: Estimated impulse response functions of expected inflation and the interest rate from the unobserved components model after a monetary policy shock normalized to raise the detrended one-year treasury bill rate by 100 basis points. The black solid line shows the response of the variables from the model allowing for sticky information, where  $\omega$  was estimated to be 0.2823. The dark and light shaded areas are the associated 68% and 90% asymptotic confidence intervals. For comparison, the dashed line shows the response of the variables from the model with no information frictions ( $\omega = 0$ ). Detailed calculations of the impulse response functions and confidence intervals are outlined in Section 4.2.





Note: The black solid line shows the cumulative impact of monetary policy shocks on the unobserved components from the model allowing for sticky information, where  $\omega$  was estimated to be 0.2823. For comparison, the dashed line shows the cumulative impact of monetary policy shocks on the unobserved components from the model with no information frictions ( $\omega = 0$ ). The shaded regions correspond with the Federal Reserve's asset purchase programs. The first of these (QE1) runs from the fourth quarter of 2008 until the first quarter of 2010. The second (QE2) runs from the fourth quarter of 2010 until the second quarter of 2011. The third of these (QE3) runs from the third quarter of 2012 until the fourth quarter of 2013.





Note: Percentage of variation in the forecast errors of the unobserved components from the state space model due to monetary policy shocks. The black solid line shows the results from the model allowing for sticky information, where  $\omega$  was estimated to be 0.2823. For comparison, the dashed line shows the results from the model with no information frictions ( $\omega = 0$ ).