Unemployment Volatility and Networks

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Abstract

I incorporate social networks into a search and matching model, allowing for congestion effects. The model predicts that the presence of network externalities increases the volatility of unemployment and other variables. I demonstrate analytically that aggregate matching functions exhibit decreasing returns to scale under certain conditions, that unemployment and matching rates have a larger response to productivity shocks, and that labour market tightness adjusts more slowly to its steady-state. Numerical simulations demonstrate that network effects can generate increases in the volatility of unemployment and matching rates, as well as increases in the autocorrelation of vacancies.

Keywords: Social Networks, Unemployment, Search and Matching.

JEL Classification: D85, E24, J64.

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1 Introduction

The use of social networks is pervasive in the matching of workers and firms. Petrongolo and Pissarides (2001) declare, "The matching function summarizes a trading technology between agents who place advertisements, read newspapers and magazines, go to employment agencies, and *mobilize local networks*[1] that eventually bring them together into productive matches." Nonetheless, there are few attempts to explicitly incorporate local networks into standard labour search models[2]. This paper models social networks in the labour market and thereby provides microfoundations for the matching function. This gives insights into several stylized facts.

First, search models of the labour market do a poor job of explaining the short-run volatility of unemployment (and other variables) in the post-war era. In fact, observed unemployment rates are an order of magnitude more volatile than a benchmark search model predicts.

Second, vacancy rates observed over the post-war era exhibit more persistence than search models predict. In particular, the observed quarterly autocorrelation of vacancy rates is larger than predicted.

I develop a model of unemployment and social networks to analyze the above facts. Unemployed workers utilize local networks to search for job openings. The resulting matching function has different properties than is often assumed in the literature. My model predicts that if the network structure is fixed then the equilibrium unemployment rates and matching rates have a larger response (in absolute value) to productivity shocks than in baseline search models. Vacancies also exhibit more sluggish transition dynamics.

When examining unemployment volatility, the results rely on variables being in steady-state, and are qualitative in nature. To check the robustness of the analytical results several numerical simulations are provided. I show that the standard deviations of several labour market variables can be increased with the inclusion of network effects. Vacancies exhibit higher autocorrelation. I conclude that network effects alone can essentially match observed unemployment volatility. However, I find that doing so produces a counterfactual unemployment-vacancy relationship, which suggests a bound on the empirical importance of network effects.

The model presented here allows for congestion effects, which occur when unemployed workers compete for jobs found though mutual (employed)

[1]Emphasis my own.

[2]There are a few exceptions discussed in the literature review section.

	u	v	v/u	p
Std. Dev.	0.190	0.202	0.382	0.020
Auto Correlation	0.936	0.940	0.941	0.878
u	1	-0.894	-0.971	-0.408
v	-	1	0.975	0.364
v/u	-	-	1	0.396
p	-	-	-	1

Table 1: U.S. Labour Market Statistics, 1951-2003 (Source: Shimer, 2005)

friends. Despite this layer of complexity, a matching function can still be derived. The matching function satisfies properties similar to those in the literature, though not identical. In particular, difficulty arises when assessing returns to scale of the matching function. The matching function only satisfies decreasing returns to scale when the unemployment rate is increased in "just the right way, because the distribution of unemployment is important.

In solving for an equilibrium, my approach relies on utilizing a *mean-field approximation*. The idea is the following: while the distribution of unemployment matters for individual matching rates, it has little impact on aggregate matching rates (on average). A consequence of this approach, combined with congestion effects, is that the aggregate number of social links in the network does not affect equilibrium unemployment, vacancies, or wages. However, the network effects still matter and congestion amplifies their impact of volatility.

The paper is organized as follows. Section 2 presents the relevant stylized facts and literature. Section 3 describes the environment and derives a micro-foundation for a network-augmented matching function. Section 4 presents the model with fixed networks and analytical results based on a mean-field approximation. Section 5 presents calibration results. Section 6 concludes. Appendix 1 contains proofs, Appendix 2 contains the general mean-field analysis, and Appendix 3 provide an example to demonstrate the accuracy of the mean-field analysis.

2 Evidence and Literature

2.1 Social Networks and Employment Transitions

There are several studies that document the use of social networks in the labour market. In a study of 2553 Quebec government workers, Langois (2007) finds that 42.7% of workers found their current job through a contact.

Erickson and Yancey (1980) finds that 57.7% of workers found their current position through a strong or weak tie. Granovetter (1983) provides a detailed survey, and argues that weak ties (acquaintances) are the primary sources of job information. Jackson (2008) provides another survey and finds that the percentage of workers who found their job through a personal contact is 23.5% in the lowest industries and 73.8% in the highest industries.

There is a large literature that supports the notion that social networks improve labour market outcomes. Laschever (2009) uses data on World War I draftees and the 1930 U.S. census to identify the impact of social networks on employment likelihood and finds that an additional employed peer increases employment likelihood by 0.8 percent. Beaman (2012) looks at the reallocation of refugees and finds that "tenured" members of the social network improve outcomes and members that are new arrivals harm employment outcomes.

Khan and Lehrer (2012) use data from a field market experiment in Cape Breton, Canada and find that although the Community Employment Innovation Project tends to increase an individual's weak ties, aggregate employment outcomes do not improve. However, those with more links tend to do better than those with fewer links. This result is consistent with our equilibrium; the mean-field approximation, combined with congestion, means that the aggregate number of links do not matter.

2.2 Unemployment and Vacancy Dynamics

There are several labour market variables that will be analyzed. The unemployment rate is the fraction of unemployed workers in the labour force, denoted u. The vacancy rate is the number of vacancies as a fraction of the labour force, denoted v. The labour market tightness is the ratio of vacancies to unemployed workers, or $\frac{v}{u}$. The number of matches per period per worker is denoted m. Finally, p is the level of worker productivity.

There is a large body of literature on the shortcomings of search models. Table 1 shows some basic properties of U.S. labour market data from 1951-2005.[3] of unemployment (0.19), and the autocorrelation of vacancies (0.94). Shimer (2005) develops a stochastic version of Pissarides (2000), calibrates the model to U.S. data, and finds that the model departs from U.S. data in important ways. In particular, predicted unemployment volatility is too low, and predicted vacancies have low (quarterly) autocorrelation compared to U.S. data. Andolfatto (1996) embeds search frictions in a real business

[3]Standard deviations are of logged variables taken from detrended data using the HP filter with smoothing parameter 10^5.

	Std. Dev.	Autocorrelation	u	v	v/u	p
m	**0.118**	0.908	-0.949	0.897	0.948	0.396

Table 2: U.S. Job Finding Rate, 1951-2003 (Source: Shimer, 2005)

cycle model and finds that volatilities in U.S. data are larger than the model can account for. Costain and Reiter (2007) investigate a search model with stochastic unemployment benefits and find a fundamental tradeoff between unemployment volatility and the impact of unemployment benefits on unemployment.

Cardullo (2010) provides a survey of the attempts to model unemployment volatility and vacancy creation in a manner that overcomes the Shimer (2005) critique. Recalibration (Hagedorn and Manovskii, 2005) and rigid wages (Hall, 2005; Pissarides, 2010) fail to satisfactorily match U.S. data. Barnichon (2012) proposes a model with endogenous productivity. Several other microfoundations have been proposed, each with varying degrees of success (see Cardullo, 2010). In the same survey, several explanations of vacancy persistence are presented.

The work presented here belongs in the microfoundation classification. I propose a mechanism (social networks) that leads to matching rate volatility and persistence, which drives volatility and persistence in unemployment and vacancies.

Another statistic that is difficult to replicate is the job finding rate m. Table 2 describes the job finding rate during 1951-2003 in the U.S. Similar to unemployment, matching rates are more volatile in the data than standard search models predict. Several studies find matching rates to be procyclical; the probability of finding a job, given the vacancy-unemployment ratio, varies positively with the business cycle. Sedlacek (2010) finds that match efficiency is procyclical and explains 26-35% of job finding rate variation.

2.3 Search Models with Network Effects

Social networks have been incorporated into models of the labour market, and provided explanations for several stylized facts. First, several authors explore negative duration dependence. Calvo-Armengol and Jackson (2004) look at static social networks and find that employment statuses are correlated across time and path-connected individuals. Bramoulle and Saint Paul (2010) take the argument further by allowing networks to evolve over time. If social ties are created at a higher rate between workers of the same employment status then the model produces duration dependence.

Some work has been done on matching functions with a network component. Fontaine (2007) uses an urn-ball framework with a series of complete networks[4]. Calvo-Armengol and Zenou (2005) uses an urn-ball matching function with regular random networks to discuss congestion in search. Galenianos (2014) utilizes a matching function in an environment with new firms and expanding firms to discuss procyclical matching efficiency. In every case the matching function exhibits decreasing returns to scale in unemployment and vacancies.

My model has both similarities and differences to Fontaine (2007) and Calvo-Armengol and Zenou (2005). The papers are similar to this one in that the resulting matching functions exhibit similar properties. Furthermore, both papers employ an urn-ball matching foundation. My paper does not restrict the network topology to complete networks or random networks. To overcome the complexity I use a mean-field approximation[5], which can be thought of as restricting the distribution of unemployment.

There are similarities and differences between my model and Galenianos (2014) as well. The resulting matching functions have similar properties. However, Galenianos (2014) abstracts from the details of the network, which is problematic if most agents have a small number of friends relative to the entire population. Furthermore, the foundation is not based on an urn-ball approach[6], and augments the standard Cobb-Douglas matching function directly. Furthermore, my model incorporates congestion effects and does not impose large neighbourhoods.

There is a more recent literature involving multiple equilibria (Tumen, 2011; Merlino, 2014; Eeckhout and Lindenlaub, 2015), which suggests that labour market variables can have larger volatility due to jumping between equiibria. The model presented here does not exhibit such multiple equilibria. Furthermore, Galeotti and Merlino (2014) and Schmutte (2015) deal with endogenous contact networks, whereas the model presented here takes networks as fixed.

3 Networks and Matching

Here I examine networks with fixed links to analyze short-run changes in

[4]A complete network is a network in which all nodes are connected. Fontaine (2007) has finite groups in which agents are connected to everyone in the same group and no one from another group.

[5]I discuss the approach in Section 4 and Appendix 2.

[6]The approach of Galenianos (2014) has firms being born, or expanding and dividing. A firm that expands accepts applications from the current employee's contacts.

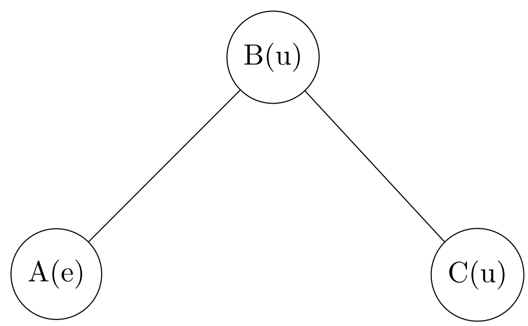

Figure 1: An example with three workers in a social network, two of which are unemployed.

unemployment, vacancies, and matching rates. First, elementary examples and definitions related to network analysis are provided. Second, I outline the modelling methodology. In particular, urn-ball matching and the telephone-line queuing process are discussed.

3.1 Networks in the Labour Market

Consider the following scenario. An unemployed agent has d^E employed friends and is filling out applications. The agent engages in random search by applying directly to firms. The agent also engages in network search by having employed friends fill out applications on his behalf.

If μ_R random search applications are filled per period and each employed friend fills out μ_N applications per period, total applications from the worker per period are $\mu_R + \mu_N d^E$. Thus, the number of applications, and consequently the rate at which an unemployed worker receives job offers, depends on a property of the network.

The following example illustrates the interaction. Figure 1 depicts a society. Agents B and C are unemployed. However, agent B has access to an employed friend. Thus, there is a total of $\mu_R + \mu_N$ applications from B and a total of μ_R from C per period.

To abstract from the problem that the distribution of firm size and vacancies over firms matter, it is assumed that all vacancies are individual firms and that employed agents randomly communicate with these firms. The details will be described when discussing the nature of vacancies.

Now suppose that employed agents can only apply for a total fixed amount of jobs, μ_N. In this case, there is competition for jobs. For example, if B was employed and A was unemployed then A and C would be competing for B's search effort. We call this phenomenon *congestion*. In particular, a worker sends applications at rate $\mu_R + \mu_N \hat{d}^E$ where $\hat{d}^E$ is a term that incorporates congestion.

To formulate $\hat{d}^E$ precisely we need some definitions. Let (L, T) be a network, where L is the labour force and T is a set of ties or relationships. Assume that network is fixed over time. A typical element of L is i and a typical element of T is (i, j).[7] The set L can be partitioned into sets U and E, which are the unemployed and employed workers, respectively. This induces a partition on the set T:

$$T_{UU} = \{(i, j) \in T | i, j \in U\}$$

$$T_{EE} = \{(i, j) \in T | i, j \in E\}$$

$$T_{UE} = \{(i, j) \in T | i \in U, j \in E\}$$

Notice that $T_{UE} = T_{EU} \subset E \times U$, $T_{UU} \subset U^2$, and $T_{EE} \subset E^2$

We will use lowercase (capital) letters to denote variables associated with unemployed (employed) workers. The set of nodes j such that $(i, j) \in T$ is referred to as i's *neighbourhood*, and is denoted n_i or N_i. The size of i's neighbourhood is called i's degree, and is denoted d_i or D_i.

A worker's set of friends of employment status $S \in \{E, U\}$ is their *S-neighbourhood*, denoted n_i^S or N_i^S. The *E-degree* is denoted d_i^E or D_i^E. Similarly one can define n_i^U, N_i^U, d_i^U, and D_i^U.

The aggregate properties of the network can be described by the *degree distribution*, denoted $f : \mathbb{N} \to [0, 1]$, which gives the proportion of workers with degree d. Similarly, the *S-degree distribution*, denoted $f^S : \mathbb{R}_+ \to [0, 1]$, described the proportion of agents with S-degree d^S.[8]

When describing a neighbouring node we will need more information than employment status. For instance, an employed friend is more valuable if he has fewer unemployed friends. It will be necessary to consider the number of k-friends a node i has as $d_i(k)$, where k is the number of employed friends they have. Similarly, we will define $d_i^E(k)$ as the number of employed friends with k employed friends that i has.

With these definitions in hand one can define $\hat{d}_i^E$. Assuming agents treat each unemployed friend equally, the rate at which an employed friend

[7] There is no distinction between (i, j) and (j, i). This is referred to as an *undirected* network.

[8] One can also describe the network structure with an adjacency matrix, A, with typical element $a_{ij} = 1$ if $(i, j) \in T$ and 0 otherwise.

j applies on behalf of an unemployed friend is $\mu_N \frac{1}{D_j^U}$. We can write the rate at which i hands out applications as

$$\hat{d}_i^E = \mu_R + \mu_N \sum_{j \in n_i^E} \frac{1}{D_j^U}$$

Before moving on, it may be useful to refer to the *degree set* $\chi \equiv \{(f(d), d)_{d \in \mathbb{Z}_+}\}$. Furthermore, we can refer to the *S-degree set* defined as $\chi^S \equiv \{(f^S(d), d^S)_{d^S \in \mathbb{R}_+}\}$.

3.2 Urn-Ball Matching

A useful approach to describing the total number of matches in a given period is with a *matching function*. Normally, matches are exogenously given as a function of the total number of searchers on both sides of the market, in this case $m(u, v)$. Furthermore, it is standard to assume that the function has several desirable properties, such as constant returns to scale.

There is a large literature that derives the matching function from first principles. One common approach is referred to as the *urn-ball method*. I utilize the urn-ball method to provide foundations for the matching function.

Consider a set of urns, V, and a set of agents U. Each agent possesses $\mu_R > 0$ balls and places each ball in a particular urn with probability $\frac{1}{|V|}$. Thus, agents pick the urn in which to drop a ball randomly with replacement.

Now suppose that each urn belongs to a different firm. Once every ball is placed in some urn the firm draws a ball from its urn at random. Some firms receive no applications and thus draw zero balls. Therefore, a firm draws a ball at random conditional on its urn containing at least one ball. If a firm draws a ball belonging to worker i then the worker gets the job. In the case of worker i getting several balls drawn he chooses the job at random.

The above environment describes a common urn-ball process. Our process is slightly different. Each employed agent also places balls in urns. An employed agent samples an urn with replacement μ_N times for each of his employed friends. The following definition allows us to discuss matching.

Definition 1 (Matches)

(i) The matching function *gives the number of matches for unemployed workers per worker as a function of the unemployment rate (u), vacancy rate (v), and period length (Δt), and is denoted by* $m(u, v, \Delta t)$.

(ii) The k-matching function *gives the number of matches for unemployed workers* $\hat{d}_i^E = k$ *as a function of unemployment, vacancies, and period length, and is denoted by* $m(u, v, \Delta t, k)$.

(iii) The matching rate *is the number of matches per worker per unit time, and is denoted* $\frac{m(u,v,\Delta t)}{\Delta t}$.

(iv) The k-matching rate *is the number of matches per worker with* $\hat{d}_i^E = k$ *per unit time, and is denoted* $\frac{m(u,v,\Delta t,k)}{\Delta t}$.

I will often suppress notation by having $m(u, v, \Delta t, k) \equiv m(\Delta t, k)$ and $lim_{\Delta t \to 0} \frac{m(u,v,\Delta t)}{\Delta t} \equiv m(u, v)$. Furthermore define

$$m_u(u, v) \equiv \frac{m(u, v)}{u}$$
$$m_u(u, v, k) \equiv \frac{m(u, v, k)}{u(k)}$$
$$m_v(u, v) \equiv \frac{m(u, v)}{v}$$

The following lemma is useful. It provides a matching function with network effects in a continuous time environment and follows from a well-known result discussed in Petrongolo and Pissarides (2001). Let $\bar{\hat{d}}^E \equiv \frac{\int_{i \in U} \hat{d}_i^E di}{u}$.

Lemma 1

Suppose that $|V|$, $|U| \to +\infty$ *where* $\frac{|V|}{|U|} = \theta < +\infty$.

If:

(i) Each unemployed agent searches randomly with intensity μ_R*,*

(ii) Each employed agent searches with intensity μ_N *on behalf of each unemployed friend, and*

(iii) When an unemployed agent sends out an application there is a probability $1 - \xi$ *that the application is destroyed,*

then when $\Delta t \to 0$ *the aggregate matching rate is*

$$m(u, v) = (\mu_R \xi + \mu_N \bar{\hat{d}}^E) u$$

The assumption (*iii*) adds uncertainty to the application process. The reason for the assumption will become clear in the next section. The uncertainty is only applied to the random search process (as opposed to the network search process). The justification is that unemployed agents send applications to a firm not knowing whether a vacancy is available. In contrast, employed agents know whether the firm is ready to hire. Our results do not depend on ξ applying to the unemployed searchers only.

The matching function has several desirable properties. First, it is linear in u. As is evident in Petrongolo and Pissarides (2001), linearity is a standard property of urn-ball matching functions in continuous time. Another desirable property is the linearity in $\bar{\hat{d}}^E$.

3.3 Telephone Line Queuing Process

There are well-established shortcomings with the urn-ball matching function. In continuous time the function does not depend on the number of vacancies.[9] Here I follow Stephens (2007) and use a *telephone-line queuing process* to endogenize ξ.

To illustrate the idea suppose that vacancies come in two types: processing, waiting. Processing vacancies are those vacancies that have been created but are unready to be filled. The justification for this is that time and effort is involved in between the decision to create a vacancy and the interview process. Waiting vacancies are those vacancies that are ready to be filled. Let $V_w \subset V$ be the set of waiting vacancies and $v_w = \frac{|V_w|}{|L|}$.

Random search occurs according to a telephone-line queuing process. Workers call a firm (uniform) randomly. If the firm has a processing vacancy it does not pick up the telephone. If the firm has a waiting vacancy it picks up the phone and a match is created. Thus, the probability that a phone call reaches a waiting vacancy is $\frac{v_w}{v}$. The matching function can be rewritten as $(\mu_R \frac{v_w}{v} + \mu_N \bar{\hat{d}}^E)u$.

Network search has an added benefit. Namely, employed workers know that a vacancy is waiting. Thus, all network applications reach a waiting vacancy[10].

To pin down $\frac{v_w}{v}$ one must discuss the determinants of v_w. If processing vacancies become waiting vacancies at rate μ_v then the inflow of waiting va-

[9]It is possible to include vacancies but then the function fails to satisfy $m(0, v) = m(u, 0) = 0$.

[10]As mentioned earlier, the results do not rely on the assumption. The difference to Proposition 1 is a matching function of $(\mu_R + \mu_N \bar{\hat{d}}^E)\xi u$.

cancies is $\mu_v(v-v_w)$. Furthermore, vacancies are being filled at the matching rate $m_u(u,v)$. Therefore, waiting vacancies evolve according to

$$\dot{v}_w = \mu_V(v - v_w) - \mu_R u \frac{v_w}{v} - \mu_N \bar{\hat{d}}^E u \tag{1}$$

the following proposition states the properties of the matching function when the those of low degree tend to be unemployed. Let $u(d)$ be the unemployment rate for those with degree d.

Proposition 1 (Steady-State Matching Function)

Suppose that $\dot{v}_w = 0$, $\mu_N > 0$, and $f(d) > 0$ for some $d > 0$.

(i) The steady state matching function is:

$$\frac{\mu_V v}{\mu_R + \mu_V \theta}\left(\mu_R + \mu_N \frac{\int_{i\in U}\sum_{j\in n_i^E} 1/D_j^U \, di}{|U|}\right)$$

(ii) Suppose the unemployment rate is increased from u to $\gamma u \equiv u'$. If $\frac{u(d)}{u} = \frac{u(d)'}{u'}$ and $U \subset U'$, then the matching function $m(u,v)$ exhibits decreasing returns to scale in (u,v).

(iii) $m(u,v)$ is strictly increasing in $\bar{\hat{d}}^E$ and v.

(iv) for large enough $\frac{m_u(u,v)}{\mu_N u}$, $m(u,v)$ is increasing in u.

The result characterizes the matching function. Part (i) gives its functional form. Part (ii) demonstrates that the matching function exhibits decreasing returns to scale, but only under certain restrictions on the distribution of unemployment. There are more workers searching, each with less intensity, which is a standard result in the literature. However, this result underlines the importance of the exact manner in which unemployment increases. One way to think of this is by swapping the employment status of $i \in U$ and $j \in E$. Standard search model predict the same number of matches. In this model, matches may increase or decrease.

Part (iii) states that the number of matches increases with v and $\bar{\hat{d}}^E$. Finally, part (iv) says that the unemployment rate always increases matches at low unemployment rates or if network effects are small. However, it is possible for unemployment to decrease the number of matches at high levels of unemployment with strong network effects. Properties (ii) and (iv), and decreasing returns to scale are common to the literature[11].

Both random and network search intensities are taken as exogenous here, whereas Stephens (2007) has endogenous search intensity. Allowing endogenous random search intensity leads to a different function form for the steady-state matching function, and raises a few issues. However, the properties of the matching function important for unemployment fluctuations are unaltered[12].

4 Model with a Fixed Network

A shortcoming of the previous section is highlighted in Proposition 1 part (ii): the stock of unemployment does not give enough information to solve the model. Here I present a search model and provide a notion of steady-state network effects.

Before moving on let λ be the rate at which workers become unemployed, called the separation rate. The unemployment rate for workers with k employed friends evolves as follows

$$\dot{u}(\hat{d}^E) = \lambda(1 - u(\hat{d}^E)) - u(\hat{d}^E)m_u(\hat{d}^E) \tag{2}$$

Average unemployment evolves according to the average of the above equation, and is the equation of interest.

The steady-state matching function of the previous section, $\frac{\mu_V v(\mu_R+\mu_N \bar{d}^E)}{(\mu_R+\mu_V\theta)}$, is taken to be fixed and non-varying. That is, $\dot{v}_w = 0$ for all t. While the main results do not rely on this, it allows for a clearer description of the impact of network effects on the standard search model.

4.1 Mean-Field Approximation

The goal of the model is to analyze the labour market dynamics. The dynamics of the model presented thus far can get very complicated. For

[11] Galenianos (2013) provides a matching function with the above properties when workers have a continuum of friends. Calvo-Armengol and Zenou (2005), with the exception of point (*ii*), derives the results with random regular networks.

[12] An earlier version of the paper contained in my thesis found that when networks and search intensity are complementary (ie. job searchers put effort into pestering friends), search costs are close to linear, and there is heterogeneity in E-degrees then search effort becomes a lot less pro-cyclical as the majority of workers are crowded out by the most connected.

instance, the evolution of S-degrees are stochastic as workers lose and gain jobs randomly.[13] I call the model of the previous section the *true model*.

To overcome these issues, I use the method employed by Bramoulle and Saint Paul (2010) and apply an approximation model called a *mean-field approximation*.[14] Let the mean-field approximation be in continuous time and allow d^E to take on any value in $\mathbb{R}_+$. The mean field approximation (i) imposes the same distribution of employment over neighbours for all workers, (ii) has the aggregate distribution of employment evolve deterministically, and (iii) has individual distributions be consistent with the aggregate distribution. Another way to think of (i) is as having a representative agent for each d, because each agent with degree d has the same E-degree.

The rest of the paper utilizes the following assumption.

Assumption 1 (Regular Networks)

Networks are regular: $d_i = D_j = d > 0$ *for all* $i \in U$ *and* $j \in E$.

The assumption places a restriction on the network topology. While the mean-field approach does not rely on this assumption, it simplifies the analysis and simulations significantly. Appendix 2 presents the general mean-field equations.

Assumption 1, combined with (i) of the mean-field approximation, allows us to simplify our matching rate. In particular, $\hat{d}_i^E = \frac{d^E}{D^U}$ for all $i \in L$. In continuous time, $m(u, v, d^E, D^U)$ is the matching rate and λ is the separation rate. The following equations describe the mean-field approximation.

$$\dot{d}^E = m_u(u, v, d^E, D^U)(d - d^E) - \lambda d^E \tag{3}$$

$$\dot{d}^U = -m_u(u, v, d^E, D^U)d^U + \lambda(d - d^U) \tag{4}$$

$$\dot{D}^E = m_u(u, v, d^E, D^U)(D - D^E) - \lambda D^E \tag{5}$$

$$\dot{D}^U = -m_u(u, v, d^E, D^U)D^U + \lambda(D - D^U) \tag{6}$$

[13] Even if the means of aggregate variables move deterministically, the individual variables are stochastic.

[14] Calvo-Armengol and Zenou (2005) overcomes these issues by randomly drawing the set of links every period. Galenianos (2013) and Galenianos (2014) overcome these issues by having neighbourhoods be infinitely large.

Equations (3) through (6) depend on the matching function, which depends on d^E and D^U. We wish to establish the steady-state of this system. Setting $\dot{D}^U = \dot{D}^U = \dot{D}^U = \dot{D}^U = 0$ gives the steady-state, which is defined by the following equations.

$$d^{E*} = \left(\frac{m_u(u, v, d^{E*}, D^{U*})}{\lambda + m_u(u, v, d^{E*}, D^{U*})} \right) d \tag{7}$$

$$D^{U*} = \left(\frac{\lambda}{\lambda + m_u(u, v, d^{E*}, D^{U*})} \right) d \tag{8}$$

Only two of the variables have been included, as they are the only ones that determine the matching rate. The steady-state has several interesting properties. First, there is no heterogeneity in E-degree or U-degree. Part of this is due to Assumption 1, which eliminates heterogeneity of degree. The other part is due to (i) of the mean-field approximation, which imposes the global distribution of employment locally. Appendix 2 considers the mean-field approximation for irregular networks, which requires an additional step of imposing global characteristics of the network topology locally.

Second, notice that $\hat{d}^{E*} = \frac{d^{E*}}{D^{U*}}$ is independent of d. In general, the impact of the total number of links is ambiguous. It depends on the distribution of unemployment and which specific links are added: more links can mean more employed friends, or more unemployed people to compete with. The mean-field approximation, along with Assumption 1, restricts the distribution of unemployment and network topology so that the two effects from giving everyone more links exactly cancel out.

Finally, as will become more clear in the next section, the mean-field approximation combined with Assumption 1 will guarantee that the matching function is decreasing returns to scale in steady-state. Again, this is a result of restricting the network topology and distribution of unemployment.

Simulations are performed in Appendix 3 to demonstrate that the mean-field approximation in this model is an accurate one. Mean-field approximations are known to be good approximations to random network models in many circumstances. Bramoulle and Saint Paul (2010) apply the approximation to a labour market model with search frictions. Jackson and Rogers (2007) applies the approximation to a network formation model. See Vega-Redondo (2007) and McComb (2004) for physical applications and general discussion. The following analysis is done with a mean-field approximation.

4.2 Equilibrium

A matching function has been derived, and now a full-fledged search and matching model may be analyzed. Here I augment Pissarides (2000) by incorporating the network effect. An equilibrium definition is given and results on volatility, persistence, and wages are stated.

One can imagine a scenario in which wages are set through bargaining and workers are heterogeneous. A non-cooperative bargaining solution can be complex and obscure the role of networks. To maintain the focus on changes in social networks (as opposed to wage setting) I determine wages with Nash bargaining as in Pissarides (2000). Furthermore, the analysis assumes that both firms and workers can observe a worker's current network position[15]. Although I believe many of the results are robust to changes in this specification the verification is beyond the scope of this paper.

Unless otherwise mentioned, I assume that waiting vacancies, S- neighbourhoods, and unemployment are in steady-state. This assumption implies that each unemployed worker of degree d_i will have the same neighbourhood composition, which makes the analysis much easier.

Let r be the (common) rate of return, b be unemployment benefits, p be productivity, $\theta = \frac{v}{u}$ be labour market tightness, and $u(d)$ be the unemployment rate of workers with degree d (defined as $u(\hat{d}^E(d))$). Notice that unemployment depends on the *total* number of friends an individual has instead of the number of employed friends. This is because the steady-state conditions pin down d_i^E and D_i^U for all i as a function of d_i.

$W_U(d)$ is the flow utility of being unemployed with degree d. $W_W(d)$ the flow utility of working with degree d. The flow utility of a vacancy is W_V. Similarly, filled jobs have flow utility depending on the type of worker hired, denoted $W_J(d)$. Because wages can be conditioned on a worker's degree value functions are too. The value of vacancies, V, does not depend on any individual worker's degree because of the uncertainty in the matching process.

The value functions[16] associated with employment status and vacancy status are

[15] This includes neighbourhood composition and degree.

[16] These are valid assuming d_i^E, D_i^U, and u are in steady-state. Otherwise the value functions have additional terms related to the changes in state-variables.

$$rW_U(d) = b + m_u(d)(W_W(d) - W_U(d)) \tag{9}$$
$$rW_W(d) = w(d) + \lambda(W_U(d) - W_W(d)) \tag{10}$$
$$rW_J(d) = p - w(d) + \lambda(-W_J(d)) \tag{11}$$
$$rW_V = -cp + \int_U m_u(d_i)(W_J(d_i) - W_V)di \tag{12}$$

for all d where $c > 0$, $p > 0$, and $b > 0$.

The value functions differ from the baseline Pissarides (2000) model in an important respect. Worker utility depends on degree d. This is because the future probability of becoming employed (potentially) depends on d. This means wages will depend on degree, which means the value of a job depends on d. Also notice the decomposition of the matching function into network and random matching components.

The steady-state conditions and Nash bargaining allow us to derive expressions for unemployment, wages, and S-neighbourhoods. To complete the model one must determine the vacancy rate v. Each vacancy is a firm and profit maximization involves decided between creating a vacancy or not. The final condition is the free-entry condition, $W_V = 0$.

The following definition is the equilibrium concept for fixed networks.

Definition 2 (Fixed Network Equilibrium)

Given a network (L, T)*, a Fixed Network Equilibrium (FNE) satisfies:*

(i) Steady-State Unemployment: $\dot{u}(d) = 0 \ \forall d$

(ii) Steady-State Neighbourhood: $\dot{d}_i^E = \dot{d}_i^U = \dot{D}_j^E = \dot{D}_j^U = 0, \forall i \in U, j \in E$

(iii) Nash-Bargaining: $w(d) = \mathit{argmax}(W_W(d) - W_U(d))^\beta (W_J(d) - W_V)^{1-\beta}$

(iv) Free-Entry: $W_V = 0$

Conditions (i), (iii), and (iv) are similar to Pissarides (2000). The main difference is that wages, and thus value functions, depend on an agent's degree[17]. Condition (ii) is a steady-state condition on S-neighbourhoods.

[17] Technically, they depend on d^E. Conditions (i)-(ii) imply that d_i^{E*} is entirely determined by d_i.

4.3 Results on Volatility and Persistence

A FNE can explain some labour market statistics due to the decreasing returns to scale matching function. First, changes in productivity, p, lead to changes in unemployment rate, u. The changes in u affect the E-neighbourhoods of unemployed workers. This leads to a lower matching rate and lower steady-state unemployment. There is a feedback effect as changes in u are reinforced by less network matching.

Second, vacancy rate v is a jump variable. Thus, changes in p lead to jumps in v. However, now u has a larger effect on v. To see this examine θ. In a baseline search model θ exhibits no persistence; exogenous changes in p lead to a one-time jump in θ. In the model presented here, exogenous changes in p lead to an initial jump in θ followed by a gradual change towards a steady-state.

When looking at the effect of p on u one must look at the direct effect and the indirect effect (through θ). The indirect effect is assessed by examining the elasticity of θ with respect to p. The next result states that the equilibrium response in θ (and u) to changes in productivity is larger with network search than with no network search.

Proposition 2 (Volatility)

Suppose the network satisfies Assumption 1 and let $\epsilon(\mu_N)_{u,(p-b)}$ be elasticity of u with respect to $p-b$ for μ_N and that $\mu_R + \mu_N d^E$ is constant. If $\mu_N > 0$ then

$$\epsilon(\mu_N)_{u,(p-b)} > \epsilon(0)_{u,(p-b)}$$

The result demonstrates that equilibrium unemployment can exhibit larger volatility. The feedback effect of losing intermediaries (employed workers) between firms and the unemployed magnifies that response of each variable. Granted the results are limited to the steady-state, if transitory dynamics are of little consequence then the result is a good approximation.

The next proposition looks at the non-equilibrium dynamics of vacancy creation. Essentially, if wage determination and free-entry ((iii)-(iv) of FNE definition) remain then θ is a function of past p and v exhibits more persistence. Let $\theta(\mu_N)$ be the tightness that satisfies $(iii)-(iv)$ at $\mu_N \geq 0$.

Proposition 3 (Persistence)

Suppose (iii)-(iv) of FNE continue to hold, $f(d) > 0$ for some $d > 0$, and at t_0 $\dot{d}^E \neq 0$ and $\dot{u} \neq 0$. Then $\dot{\theta}(\mu_N) \neq 0$ for $t > t_0$ implies $\mu_N > 0$.

The result demonstrates that network effects lead to a non-zero $\dot{\theta}(\mu_N)$. The labour market tightness in the model with $\mu_N = 0$ is not a function of flow variables. Now changes in d^E and u have an impact on θ by steadily increasing (decreasing) the matching efficiency when p increases (decreases).

5 Numerical Simulations

A numerical simulation is performed to see if non-steady-state dynamics lead to different results. The model is calibrated to match certain moments from the U.S. data from 1951-2003. Here we use the dynamic version of the model, as in Shimer (2005). Equilibrium tightness is implicitly defined by:

$$\frac{r+\lambda+\gamma}{m_v(\theta_{p,d^E}, d^E)} + \beta\theta_{p,d^E} = \frac{(1-\beta)(p-z)}{cp} + \gamma E_p[1/m_V(\theta_{p',d^{E'}}, d^{E'})] \quad (13)$$

where γ is the rate at which p is exogenously changed. Notice that unlike the previous literature, tightness depends on the current variable d^E and future variable $d^{E'}$. Solving the system is straightforward for many computer packages. However, the equation must be solved at each iteration because d^E changes, increasing the computation time.

5.1 Calibration

The calibration strategy follows Shimer (2005) fairly closely[18]. The addition of U.S. data beyond 2003 changes the aggregate statistics very little. Productivity follows a discretized Ornstein-Uhlenbeck Process to match the volatility present in U.S. data, though the state space is smaller due to computational limitations. The unemployment utility, $b = 0.4$, is chosen within the range of unemployment benefits as a percentage of mean income.

The bargaining power, $\beta = 0.72$, is chosen to satisfy the Hosios Condition in a standard search model. Unlike Shimer (2005), the matching function used here has a non-constant elasticity of vacancies. Nonetheless, the chosen bargaining weight is close to the average elasticity of vacancies.

The separation rate, $\lambda = 0.09$, is chosen to be close to that observed in U.S. data. The parameters left are used to target relevant means, varying the relationship between μ_R and μ_N across simulations. Table 3 summarizes the choices across the three simulations.

[18] Hagedorn and Manovskii (2008) take issue with the calibration strategy. These criticisms do not change the main message that networks can explain some unemployment volatility.

	No Network Effect	Large Network effect	Small Network Effect
λ	0.09	0.09	0.09
c	0.32	0.32	0.28
β	0.72	0.72	0.72
b	0.4	0.4	0.4
d	-	100	100
$\frac{\mu_V}{\mu_R}$	1/4	1/6	1/6
$\frac{\mu_N}{\mu_R}$	0	1/18	1/1000
μ_R	6.3	9.2	100

Table 3: Calibrated Variables

Model		u	v	θ	m	p
No NE	Std. Dev.	0.012	0.010	0.017	0.013	0.017
		(0.002)	(0.001)	(0.002)	(0.002)	(0.002)
Large NE	Std. Dev.	0.165	0.154	0.016	0.180	0.017
		(0.005)	(0.002)	(0.003)	(0.005)	(0.002)
Small NE	Std. Dev.	0.017	0.010	0.018	0.019	0.017
		(0.004)	(0.002)	(0.002)	(0.005)	(0.002)

Table 4: Standard deviations for different models.

We target mean unemployment at 0.056, and mean matching rate at 1.5. Because we measure vacancies as an index, we have the freedom to target θ to 1.25 without loss of generality. Finally, we keep μ_N and μ_R within an empirically reasonable range across our second two simulations. We use the survey of Jackson (2008) to obtain targets for the fraction of jobs found through a network. The small network effect has matching through networks on average accounting for approximately $\frac{3}{11}$ (or 27.3%) of matches, whereas the large network effect account for $\frac{3}{5}$ (or 60%) of all matches on average.

Because the data is filtered, the network effect will be filtered out if it operates at a quarterly frequency. Therefore, we run simulations at a rate of 15 times more frequent than a quarter. Intuitively this means that unemployed workers talk to employed friends roughly five times per month, allowing feedback effects to work between quarters. The data is then aggregated by capturing every 15th observation.

5.2 Results

The results of the no network effect model reproduce Shimer (2005) rel-

	No Network	Large Network	Small Network
v Autocorrelation	0.600	0.944	0.622
	(0.076)	(0.079)	(0.076)
corr(u,v)	-0.658	0.944	-0.181
	(0.064)	(0.066)	(0.064)

Table 5: Autocorrelation for v and contemporaneous correlation of u and v.

atively accurately, despite using a different matching function and slight different productivity process specification[19]. Table 4 shows the standard deviation (from an HP filtered trend) and autocorrelation across models for no network effect, large network effect, and small network effect. The statistics are averaged across 1000 model simulations each, with standard errors in brackets.

Unemployment and matching rates are impacted most by network effects, increasing by 1.5 to 4 times. Vacancies are affected somewhat, roughly doubling in volatility and exhibiting a small increases in autocorrelation.

Table 5 illustrates two important results. First, network effects increase the autocorrelation of vacancies somewhat. The increase is small, but highlights the role networks play as a propagation mechanism.

However, adding network effects disconnects the unemployment-vacancy relationship. Table 5 also shows how unemployment is contemporaneously correlated with vacancies. The correlation is highly negative in the data, low and negative with small network effect, and positive with large network effects.

The positively sloped Beveridge curve suggests a bound on network effects. Much of the correlation can be regained by setting low network effects and artificially increasing the volatility of θ. Therefore, a realistic model will likely include network effects together with mechanisms that increase θ.

6 Conclusion

The role of network effects in the labour market has yet to be fully explored. I contribute to the literature by developing a model with fixed networks and evaluate the impact of network effects on the volatility and persistence of important labour market variables. I characterize the equilibrium and find that the existence of network effects increases the volatility of unemployment, and matching rates. Networks also impact the propagation of

[19] The state-space is coarser due to computational constraints.

exogenous changes in productivity.

Our results motivate the further investigation of network effects in search and matching models of the labour market. First, the result of including network effects together with other mechanisms for increasing volatility, such as rigid wages or endogenous productivity, has yet to be explored. Perhaps the inclusion of other mechanisms, together with network effects, can produce observed volatility and maintain a realistic unemployment-vacancy relationship.

Second, the model presented here takes the link structure as given. A model of endogenous link formation may yield interesting dynamics. I leave these issues for future research.

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8 Appendix A: Proofs

8.1 Proof of Lemma 1

When workers apply randomly and independently to vacancies we have: # of matches = # of vacancies × the probability a vacancy gets filled.
Rewrite this as

$$M(u, v, \chi^E) = |V| \times p$$

where $M(u, v, \chi^E)$ is the total number of matches as a function of the unemployment rate, vacancy rate, and the E-degree set. Whether a particular vacancy gets filled or not is a binary random variable and we can write $p = 1 - q$ where $q =$ the probability that all unemployed workers apply elsewhere.
If $\kappa(\hat{d}^E) = \mu_R \xi + \mu_N \hat{d}^E$ is the "search intensity" of an unemployed worker with $\hat{d}^E$ help from employed friends, then (because workers randomly and independently apply)

$$q = \Pi_{d^E=0}^{d^E_{max}} q(d^E) = \Pi_{\hat{d}^E}(1 - \frac{1}{|V|})^{\kappa(\hat{d}^E)|U(\hat{d}^E)|}$$

where $|U(\hat{d}^E)|$ is the number of unemployed workers with $\hat{d}^E$ and $q(\hat{d}^E) \equiv 1 - \frac{1}{|V|})^{\kappa(\hat{d}^E)|U(\hat{d}^E)|}$ is the probability that all unemployed workers with $\hat{d}^E$ do not apply to the vacancy. Therefore,

$$\frac{M(u, v, \chi^E)}{|L|} = v(1 - \Pi_{\hat{d}^E}(1 - \frac{1}{v|L|})^{\kappa(\hat{d}^E)u(\hat{d}^E)|L|})$$

where $|L|$ is the size of the labour force.
Taking $|L| \to +\infty$ for all $\hat{d}^E$, and letting v, $\frac{v}{u} \equiv \theta$, and $\frac{u(\hat{d}^E)}{u} \equiv \alpha(\hat{d}^E)$ be constant and finite for all $\hat{d}^E$ we get

$$m(u, v, \chi^E) = v(1 - \Pi_{d^E} e^{\frac{-\kappa(\hat{d}^E)\alpha(\hat{d}^E)}{\theta}})$$

which can be rewritten as

$$m(u, v, \chi^E) = v(1 - e^{\frac{-\bar{\kappa}}{\theta}})$$

where $\bar{\kappa}$ is the average over unemployed workers and $m(u, v, \chi^E)$ is matches per worker.
To find the continuous time version, let search intensity be $\kappa(\hat{d}^E)\Delta t$ and look for matches per worker per unit time:

$$\frac{m(u, v, \chi^E)}{\Delta t} = \frac{v}{\Delta t}(1 - e^{-\frac{\bar{\kappa}\Delta t}{\theta}})$$

Applying L'Hopital's rule gives

$$\lim_{\Delta t \to 0} \frac{m(u, v, \chi^E)}{\Delta t} = \bar{\kappa} u$$

■

8.2 Proof of Proposition 1

The proof has two steps: impose $\dot{v}_w = 0$ and solve for the steady-state matching function to get (i), and then show $(ii) - (iv)$ are true.

Step 1: First I derive the aggregate matching function as a function of the network. Setting $\dot{v}_w = 0$ yields a steady-state fraction of waiting vacancies $\frac{v_w}{v} = \frac{v - \mu_N \bar{d}^E u}{\mu_R u + v}$. I restrict attention to values where the fraction is non-negative.

The total number of matches (over a small interval) is $m(u, v, \bar{d}^E) = \mu_R u \frac{v_w}{v} + \mu_N \bar{d}^E u$. Notice the decomposition into a random search and network search component. Rearranging gives the aggregate matching function as

$$\begin{aligned} m(u, v, \bar{d}^E) &= \mu_R u \frac{v_w}{v}^* + \mu_N \bar{\hat{d}}^E u \\ &= \frac{\mu_R u(\mu_V v - \mu_N \bar{\hat{d}}^E u)}{\mu_R u + \mu_V v} + \mu_N \bar{\hat{d}}^E u \\ &= \frac{\mu_V v u}{\mu_R u + \mu_V v}(\mu_R + \mu_N \bar{\hat{d}}^E) \\ &= \frac{\mu_V v(\mu_R + \mu_N \bar{\hat{d}}^E)}{\mu_R + \mu_V \theta} \end{aligned} \tag{14}$$

One gets the matching function by realizing that $\bar{\hat{d}}^E = \frac{\int_{i \in U} \sum_{j \in n_i^E} 1/D_j^U di}{u}$

Step 2:

To show (ii) let $\gamma > 1$ and $\bar{\hat{d}}^e(d)$ be the average $\hat{d}_i^E$ over those with degree d.

$$
\begin{aligned}
m(\gamma u, \gamma v) &= \frac{\mu_V(\gamma v)}{\mu_R + \mu_V \theta}(\mu_R + \mu_N \frac{\sum_d f(d)u(d)\bar{\hat{d}}^{E'}(d)}{u}) \\
&< \frac{\gamma \mu_V v}{\mu_R + \mu_V \theta}(\mu_R + \mu_N \frac{\sum_d f(d)u(d)\bar{\hat{d}}^{E}(d)}{u}) \\
&= \gamma m(u, v)
\end{aligned}
\tag{15}
$$

The step with the inequality comes from the fact that $\frac{f(d)u(d)}{u}$ is constant, and $\bar{\hat{d}}^{E'}(d) \geq \bar{\hat{d}}^{E}(d)$ for all d with a strict inequality for some.

To show (iii) one need only differentiate $m(u, v)$ with respect to v and $\bar{\hat{d}}^{E}$ respectively. The proof is sufficiently trivial to be left to the reader.

To show (iv) one need only take the derivative of $m(u, v)$ with respect to u. Equivalently, we look at each component of the matching function.

$$
\begin{aligned}
\frac{\partial m(u,v)}{\partial u} &= \frac{(\mu_R + \mu_N \bar{\hat{d}}^{E})\mu_V v}{(\mu_R + \mu_V \theta)^2}\left(\frac{\mu_N v}{u^2}\right) + \left(\frac{\mu_V v \mu_N}{(\mu_R + \mu_V \theta)}\right)\left(\frac{\partial \bar{\hat{d}}^{E}}{\partial u}\right) \\
&= \left(\frac{m(u,v)}{u}\right)^2 \left(\frac{1}{(\mu_R + \mu_V \bar{\hat{d}}^{E})}\right) + \left(\frac{\mu_N m(u,v)}{\mu_R + \mu_N \bar{\hat{d}}^{E}}\right)\left(\frac{\partial \bar{\hat{d}}^{E}}{\partial u}\right) \\
&= \frac{\mu_N m(u,v)}{(\mu_R + \mu_N \bar{\hat{d}}^{E})}\left[\frac{m(u,v)}{\mu_N u^2} + \frac{\partial \bar{\hat{d}}^{E}}{\partial u}\right]
\end{aligned}
\tag{16}
$$

This term will be positive or negative depending on the size of the parameters. If $\mu_N u$ is large enough then, because the second term is negative, the derivative is negative.

■

8.3 Proof of Proposition 2

The proof has two steps. First, the equilibrium, described in Definition 2, is shown to be reducible to two equations. Second, given these equations (displayed as (20) and (21) below) I can calculate $\epsilon_{u,p-b}$ and $\epsilon_{\theta,p-b}$. The proof is done for $\frac{\mu_V}{\mu_R} = 1$, though the results are not sensitive to the restriction.

Step 1: The FNE conditions establish several equations. Substituting in $\hat{d}^{E*} = \frac{1-u}{u}$ into the matching function reduces the equilibrium conditions to the following equations. Note that in equilibrium each agent with degree d will have the same d^{E*}. I will suppress the $*$ notation. The remaining conditions are

$$0 = \lambda(1-u) - \frac{\theta}{1+\theta}(\mu_R + \mu_N \frac{(1-u)}{u})u \tag{17}$$

$$0 = (1-\beta)(W_W(d) - W_U(d)) - \beta(W_J(d) - W_V) \tag{18}$$

$$0 = p - \bar{w} + \frac{pc(r+\lambda)(1+\theta)}{\mu_R + \mu_V \frac{(1-u)}{u}} \tag{19}$$

where $\bar{w}$ is the average wage. Equation (17) is the steady-state imposed on (2) for $k = d$, with d^{E*} substituted in, (18) is the (rearranged) first-order condition from Nash bargaining given by (iii) in Definition 2, and (19) is the result of imposing the free-entry condition (stated as (iv) from Definition 2).
The above equation are analogous to the equations of Pissarides (2000). Equation (17) imposes steady-state unemployment, equation (18) is the first-order condition for Nash bargaining, and equation (19) is the job-creation equation. Notice that, although equations that depend on d, this will fall out due to Assumption 1 and imposing the mean-field approximation[20]
The system is reducible to the following two equations:

$$0 = \lambda(1-u) - \frac{\theta}{\mu_R+\theta}(\mu_R + \mu_N \frac{(1-u)}{u})u \tag{20}$$

$$\frac{(1-\beta)(p-b)}{c} = (r+\lambda)\frac{(\mu_R+\theta)}{(\mu_R + \mu_N \frac{(1-u)}{u})} + \beta\theta \tag{21}$$

Step 2: Let $\eta_\theta \equiv \frac{\partial m_u}{\partial \theta}\frac{\theta}{m_u}$ and $\eta_u \equiv \frac{\partial m_u}{\partial u}\frac{u}{m_u}$ be elasticities. Notice that $\frac{\mu_V}{\mu_R}$ constant implies that η_θ does not vary with μ_N or μ_R.
Taking the total derivative of the system one can solve for the elasticities. Let η_x be the elasticity of m_u with respect to x.

[20] In the case of a general degree distribution with n types, there is a system of $2n+1$ equations, because (19) just depends on the average wage.

$$\epsilon_{u,p-b} = \frac{-(1-\beta)(p-b)\eta_\theta m_u}{c\theta[(m_u+\lambda)((r+\lambda)(1-\eta_\theta)+\beta)+\eta_u m_u((r+\lambda)(1-2\eta_\theta)+\beta)]} \tag{22}$$

$$\epsilon_{\theta,p-b} = \frac{(\lambda+m_u(1+\eta_u))}{\eta_\theta m_u}|\epsilon_{u,p-b}| \tag{23}$$

Increasing μ_N from zero to a positive number decreases η_u from zero to a negative number, keeping $(\mu_R + \mu_N(1-u)d)$ constant. In this case the absolute value of (22) increases as proposed.
The absolute value of (23) changes in an ambiguous way.

■

8.4 Proof of Proposition 3

The proof involves keeping the job creation equation and wage equation from the previous proof and taking the time derivative of θ. The idea is that the network effect, which shows up in the matching function, will create persistence in changes in p. Totally differentiating the job creation equation (19) gives:

$$\left(\frac{(r+\lambda)}{E[\mu_R+\mu_N\hat{d}^E]}+\beta\right)\frac{\partial\theta}{\partial t} = \frac{(r+\lambda)(\mu_R+\theta)}{E[\mu_R+\mu_N\hat{d}^E]^2}\mu_N E[\frac{\partial\hat{d}^E}{\partial t}] + \frac{\partial\Xi}{\partial t} \tag{24}$$

where I use $E[]$ to represent averages. With Assumption 1 these expectations go away. The $\frac{\partial\Xi}{\partial t}$ is the term that captures the fact that the firm is forward looking and takes the change in d^E into account (see equation (13) in Section 5), which means $\frac{\partial\Xi}{\partial t} = 0$ in steady-state. It is clear that $\frac{\partial\theta}{\partial t} > 0$ requires a network effect $\mu_N > 0$.

■

8.5 Appendix 2: General Mean-Field Equations

Writing down the equations requires more notation, which is defined below. The following equations describe the mean-field approximation for all d:

$$\dot{d}^E(d,\chi^E) = m_u(u,v,\chi^E)(d - d^E(d,\chi^E)) - \lambda d^E(d,\chi^E) \tag{25}$$

$$\dot{d}^U(d,\chi^E) = -m_u(u,v,\chi^E)d^U(d,\chi^E) + \lambda(d - d^U(d,\chi^E)) \tag{26}$$

$$\dot{D}^E(D,\chi^E) = m_u(u,v,\chi^E)(D - D^E(D,\chi^E)) - \lambda D^E(D,\chi^E) \tag{27}$$

$$\dot{D}^U(D,\chi^E) = -m_u(u,v,\chi^E)D^U(D,\chi^E) + \lambda(D - D^U(D,\chi^E)) \tag{28}$$

where $m_u(u,v,\chi^E)$ is the matching rate of an unemployed agent and λ is the separation rate. Equations (25) through (28) depend on χ^E, in particular d^E, D^U, and d, the number of total friends. The set χ^E matters because agents with more employed friends (who themselves have few unemployed friends) will become employed faster.

Notice that $d^U(d,\chi^E) = d - d^E(d,\chi^E)$. The first term of equation (25) describes the number of U-neighbours that gain employment whereas the second term of equation (25) describes the number of E-neighbours that lose jobs. Therefore, there is no net loss or gain in total ties.

The steady-state for equation (25) is defined by $d^{E*}(d,\chi^{E*}) = \frac{m_u(u,v,\chi^{E*})}{\lambda + m_u(u,v,\chi^{E*})}d$. However, the total number of employed friends is given by

$$d^{E*}(d) = \sum_{\alpha} \hat{f}(\alpha) d^{E*}(\alpha,\chi^{E*})$$

and unemployed friends by

$$D^{U*}(d) = \sum_{\alpha} \hat{f}(\alpha) D^{U*}(\alpha,\chi^{E*})d$$

I have introduced new notation: $\hat{f}(\alpha)$ is the *neighbour degree distribution*, which gives the number of friends with degree α.

$$d^{E*}(d) = \left(\sum_{\alpha} \hat{f}(\alpha)\frac{m_u(u,v,\chi^{E*})}{\lambda + m_u(u,v,\chi^{E*})}\right)d \tag{29}$$

$$D^{U*}(d) = \left(\sum_{\alpha} \hat{f}(\alpha)\frac{\lambda}{\lambda + m_u(u,v,\chi^{*})}\right) \tag{30}$$

Notice that $m_u(u,v,\chi^{E*}) = \frac{\mu_V\theta}{\mu_V + \mu_R\theta} \times (\mu_R + \mu_N \sum_{\alpha} \frac{d^{E*}(\alpha)}{D^{U*}(\alpha)})$. Normally, a mean-field approximation would impose $\hat{f}(d)$ to be the average proportion of friends with degree d. This allows for a wide-range of network topologies. For example, Poisson random networks would impose $\hat{f}(d) = f(d)$. Poisson random graphs have many short-comings. The *paradox of friendship* states that the average degree of a neighbour exceeds the average degree over all agents; friends are more popular on average. One can correct this by setting $\hat{f}(d) = \frac{f(d)d}{d}$ as in the configuration model outlined in Newman (2010).

Note that because the network is fixed the fraction $\hat{f}(d)$ does not depend on the unemployment rate or vacancy rate. Furthermore, notice that $\hat{f}(d) = 1$ when the network is regular.

9 Appendix 3: Mean-Field Approximation of Transitional Dynamics

To demonstrate the accuracy of a mean-field approximation, a simulation is performed. An adjacency matrix A defines a network, where element a_{ij} is 1 if $(i, j) \in T$ and 0 otherwise. A random $n \times n$ adjacency matrix is generated in which each agent has 100 friends.

A time series is generated with constant p for the true model, and its mean-field approximation. Figure 2 plot the unemployment rate (vertical axis) against time (horizontal axis). The comparison of the transition dynamics for unemployment in the two models demonstrates that the mean-field analysis approximates the true model well.

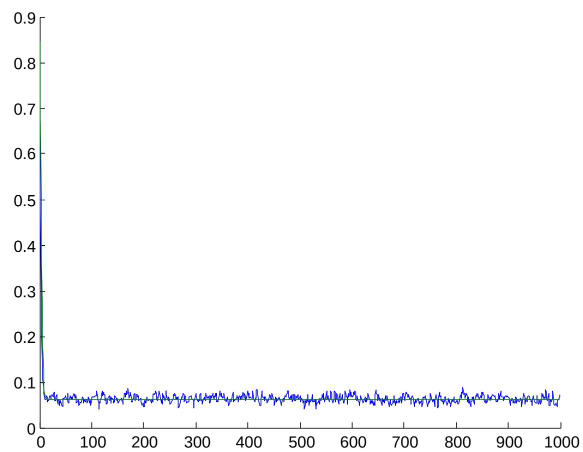

Figure 2: True Model (blue) vs. Mean-field Approximation (green)