Productivity Growth and the Future of the U.S. Saving Rate

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Productivity Growth and the Future of the U.S. Saving Rate*

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Abstract

Over the last half century, the saving rate in the United States exhibited significant medium-run variations. In this paper, I examine whether a general equilibrium model that allows for shifts in the rate of total factor productivity growth rate can account for these variations. The model matches the medium-run variations in the U.S. saving rate, and establishes a link between episodes of productivity slowdowns or accelerations and the saving rate—two concepts that have often been treated in isolation. I also use population projections and productivity forecasts to chart the future of the U.S. saving rate. Finally, I consider an extended version of the model, which treats housing as an input into the production of non-market goods, and explore the quantitative influence of housing on the consumption–income ratio.

Keywords: consumption–income ratio; saving rate; medium-run; productivity growth; U.S.
JEL Classification: E2

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1 Introduction

Over the last quarter century, the growth rate of consumption in the U.S. has far outstripped the growth rate of its GDP. Figure 1 shows personal consumption expenditures as a percentage of GDP since 1952. Both the actual and Hodrick-Prescott filtered series exhibit an upward trend, especially after the 1970s. A more familiar restatement of this trend is the declining personal saving rate in the U.S. In this paper I argue that medium-run swings in the productivity growth rate help account for this rising consumption-to-income ratio.[1]

In this paper, I examine whether a general equilibrium model, which allows for shifts in the rate of productivity growth rate, can account for these variations. I find that the model matches the broader medium-run variations in the U.S. consumption–income ratio, and establishes a link between episodes of productivity slowdowns or accelerations and the saving rate—two concepts that have often been treated in isolation.[2]

I also find that the rate of return to capital over the last 50 years has not been constant, and that the model tracks the changes reasonably well. As such, these findings provide support for the intertemporal substitution motive—a core classical theme in macroeconomics—as a key driver of the medium-run swings in the observed consumption-to-income ratio in the U.S.

More recently, with the run-up and subsequent collapse in housing prices, the ("low") saving rate in the U.S. has become a focal point of economic policy debates. Within this context, housing poses conceptual challenges, and raises measurement issues. From a measurement standpoint, conventional national income accounting treats housing (imputed rent) as a final personal consumption expenditure. In this paper I exclude housing from consumption expenditures. Instead, in an extension of the baseline model, I model housing as an input into the production of non-market goods, and explore quantitatively the indirect influence of housing on the consumption–income ratio. Overall, relative to the baseline model without housing, the extended model matches the actual data better, but the extended model still fails to account for the persistently high consumption–income ratio in the 2000s.

[1]In the quantitative sections of this paper I focus on the ratio of personal consumption expenditures to market income, which excludes gross housing value added as reported in the national income and product accounts (NIPA), and consider a closed economy. In an appendix available from the author, I discuss a range of alternative expressions of the consumption-income ratio that differ in terms of their treatment of the foreign and government sectors, and durable goods. In all these cases, I document a marked increase in the consumption-to-income ratio at least since the early 1980s. So, the differences across business, foreign, government, and personal savings are not driving the trend shown in Figure 1. See also Parker (2000) for an extensive discussion.

[2]For instance, according to the Bureau of Labor Statistics data, the business sector multifactor productivity growth rate was 2.77 percent during the period 1952–1973 ("Golden Age"), 0.82 percent between 1973–1995 ("slowdown"), and 2.09 percent between 1995–2006 ("resurgence").

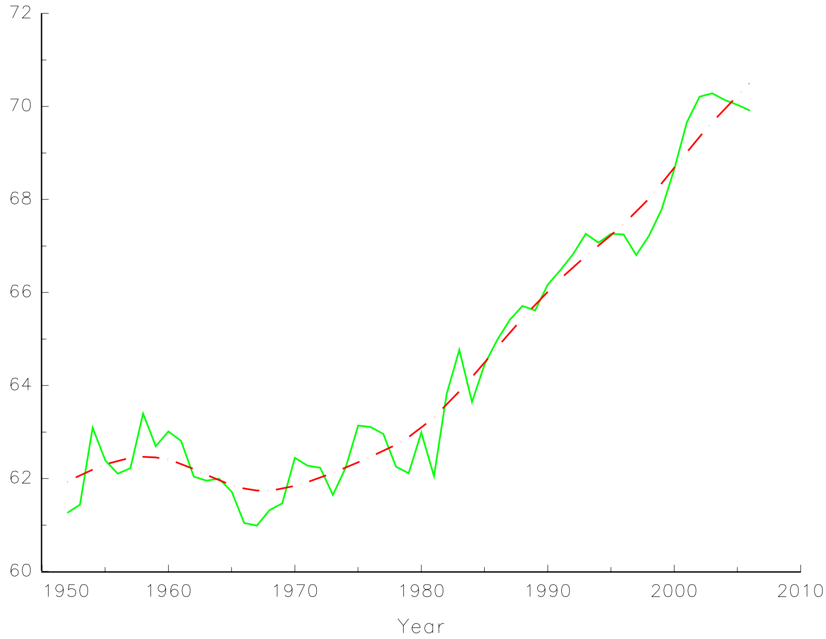

Figure 1: Personal consumption expenditures as a percentage of GDP, United States, 1952–2006
Source: U.S. Department of Commerce, Bureau of Economic Analysis.
Notes: The solid line is the actual data and the dashed line is the Hodrick-Prescott filtered data with the smoothing parameter set equal to 100.

Along a balanced growth path, the time paths of consumption and income are typically determined by a *unique* long-run growth rate of the productivity factor, and both consumption and income grow at constant rates. Moreover, since the consumption-income ratio is bounded between zero and one, they must also grow at identical rates. Hence, in a deterministic neoclassical growth model, the consumption-income ratio is constant.[3] This paper, on the other hand, accounts for the rising consumption-income ratio by appealing to shifts (or "waves") in the growth rate of total factor productivity, of the nature identified by Gordon (2004), among others. I also consider the distinction between *perceived* and actual long-run productivity growth rate. I compare the consumption–income ratios based on real-time forecasts of productivity growth with those based on currently available revised data, and find significant differences.[4]

[3]In quantitative implementations, such a ratio is sometimes called a "calibration target." When the economy is approaching to its balanced-growth path, the consumption-income ratio need not be constant.

[4]While consumption-output ratio is an appropriate calibration "target" in general equilibrium models, partial equilibrium permanent income and life-cycle models of consumption point to a slightly different but an analogous ratio—the ratio of personal consumption to personal wealth, which is equal to human plus non-human wealth. In these models, households consume out of their permanent income, which is typically the annuity value of personal wealth, and thus target a long-run consumption-wealth ratio (see Merton, 1969; Campbell and Mankiw, 1989; Deaton, 1992, chapter 3; Carroll, 2001). This ratio has recently been interpreted as a long-run relationship among

The rest of the paper is organized as follows. Section 2 discusses the existing explanations for the rising consumption–income ratio in the U.S. Section 3 presents a baseline growth model. Section 4 presents the simulation results for the baseline model, and discusses the quantitative significance of medium-run swings in productivity growth rate as a key driver of the consumption–income ratio. It also presents results from several extensions of the baseline model, including a model in which home production uses housing capital. Section 5 contains concluding remarks. A technical appendix and a data appendix complement the paper.

2 Existing explanations

In the recent literature, most of the attention has focused on five possible complementary explanations of rising consumption–income ratio: capital gains, financial innovations, demographic transition, cohort effects, and low interest rates. There is, at best, mixed evidence on the quantitative significance of each of these sources in accounting for the decline in the saving rate. In what follows I review each of these explanations and the evidence. (See Parker (2000) for a survey.)

Capital gains.—According to this explanation households have perceived the capital gains in booming stock and housing markets recorded since the mid-1990s as *permanent* (which is the component of changes in wealth that matters the most for consumption decisions), and this wealth effect has culminated in a higher consumption–income ratio (Congressional Budget Office, 2006; Council of Economic Advisors, 2006; Steindel, 2007). One practical difficulty with the capital gains explanation is the large uncertainty concerning the permanent component of changes in asset prices. The consensus view in the literature (e.g., Lettau and Ludvigson, 2004) suggests that even sizeable variations in non-human wealth typically lead to small changes in consumption, because shocks to wealth have a large *transitory* component. So, for this channel to account for the increase in the consumption–income ratio, households must have imputed a large permanent component to run-ups in equity and housing prices between 1995 and 2006.[5]

Another difficulty with the capital gains explanation is its timing: the decline in the saving rate started *before* the run-up in asset prices.[6] Moreover, the rise in the consumption–income

consumption expenditures, after-tax labor income and household net worth (Lettau and Ludvigson, 2004).

[5]Recent studies also suggest that, over the last 45 years, consumption–wealth ratio in the U.S. has gone through fundamental structural changes, which makes it difficult to ascertain the permanent component of changes in wealth. See Koop et al. (2005), Hang and Lee (2006), Rudd and Whalen (2006), and Alexandre et al. (2007).

[6]In fact, the evidence suggests that the wealth effect is more closely related to capital gains in the housing

ratio went on unabated despite large fluctuations in asset prices and wealth-to-income ratio. Therefore, while the capital gains channel theoretically holds the promise of accounting for rising consumption–income ratio, available evidence poses serious challenges to this explanation.

Financial innovations.—This explanation is based on the argument that the combination of rapid innovation and deregulation in the financial sector has allowed households to borrow against their human wealth, facilitated annuitization of non-human wealth, and consequently reduced precautionary savings (Carroll, 1992). These developments together with the changing norms of consumption have lead to a rise in the consumption–income ratio (Cynamon and Fazzari, 2008). While plausible, general equilibrium consequences of this explanation remains largely unexplored within a quantitative framework.[7]

Demographic transition and cohort effects.—One of the early explanations advanced for the declining saving rate in the U.S. suggests that as the U.S. population has aged, the fraction of the population with higher propensities to consume has increased, leading to a rise in the aggregate consumption–income ratio. However, it is difficult to empirically account for the declining saving rate through demographic transition (age affects) alone, and in none of the studies reviewed by Parker (2000) demographic transition stands out as a contender. On the other hand, there is mixed evidence for cohort effects. Using micro level data, while Bosworth, Burtless, and Sabelhous (1991) find that the saving rate of all age groups have declined uniformly over time, Gokhale, Kotlikoff, and Sabelhous (1996) attribute the decline in the overall saving rate entirely to the rise in the consumption propensities of older age groups. Recently, Doepke and Schneider (2006) document the dramatic decline in the household sector's net *nominal* asset position at least since the 1990s, and attribute it to higher nominal debt held by the young and lower nominal savings by the old.[8]

Low interest rates.—This explanation attributes the rise in consumption–income ratio to the intertemporal substitution motive.[9] In general, a higher real interest rate stimulates saving, and

market, which started around 2001, rather than to those in the stock market (Borsworth et al., 1991; Donihue and Avramenko, 2007).

[7]Using a partial equilibrium framework Parker (2000, pp. 333–334) concludes that financial innovations are unlikely to account for the large increase in the consumption–income ratio.

[8]Gokhale et al. (1996) attribute the declining saving propensity of the elderly to the introduction of medicare benefits, generous transfers from young to old generations, and improved annuitization facilities—a form of financial innovation. Doepke and Schneider (2006) also document the changes that has occurred in the duration and composition of nominal assets since the 1980s. These are also suggestive of the consequences of financial innovation—especially in the mortgage market—as a complementary source of falling saving rate.

[9]See, e.g., Parker (2000, pp. 358–362). Bosworth et al. (1991, pp. 200–201) allude to the interest rate channel.

reduces consumption–income ratio.

Figure 2 shows the real interest rate in the U.S. since 1952 using both a benchmark bond yield (panel a), and direct measures of rate of return to capital (panel b). The data in panel (a) are perhaps the most commonly used measures of the real interest rate. They show the familiar pattern that the real yields were low in the 1950s and 1960s, rose over the 1970s, reached to their peak in 1982, and have been declining since then. Based on these data alone one would be skeptical about the intertemporal substitution channel: compared to the period from 1952 to 1980, the real interest rate has on average been higher since 1980, and yet the consumption–income ratio has consistently been above that of the 1952–1980 average.

However, Figure 2 panel (b) presents a different perspective on the intertemporal substitution motive. It shows two measures of pre-tax rate of return to capital: (i) rate of return to tangible assets in the nonfinancial corporate sector computed by Poterba (1998), and, (ii) rate of return to fixed assets in the corporate sector. The two series are slightly different in magnitude, because Poterba's asset definition is broader and excludes the financial sector. However, they both exhibit identical trends. Panel (b) shows that there has been a mild but discernible *decline* over time in the rate of return to capital, coupled with medium-run swings.

Below I argue that the movements in the real rate of return to capital are more relevant for understanding the joint behaviour of (model-based) consumption and income variables. More importantly, a satisfactory understanding of the consumption–income ratio has to consider *causes*, as well as consequences of changes in the real interest rate. To capture these linkages at the macroeconomic level, I next consider a general equilibrium setup.[10]

3 The model

The model I use to examine the simultaneous determination of the rate of return to capital and the consumption–income ratio is the neoclassical growth model. I refer the reader to Gomme and Rupert (2007) for a recent exposition of this framework in a business-cycle context. In

However, their arguments are based on a life-cycle model of consumption, in which a *rise* in the interest rate reduces saving of the older generations only. While the saving rate of elderly households have declined (although remains positive), they don't find sufficiently large differences in saving rates by age for the real interest rate channel to have an appreciable effect on the overall saving rate. Cynamon and Fazzari (2008), on the other hand, claim that "no wealth effect of increasing asset prices, no substitution effect of *lower* interest rates" could explain the recent rise in both consumption and debt.

[10]In fact, in their concluding remarks Bosworth et al. (1991) note that their "result suggests that the decline in saving must involve one or more factors that affect the vast majority of households uniformly. In seeking an explanation for the drop in saving, we are thus drawn back to macroeconomic factors, rather than the demographic and microeconomic determinants that many economists currently find so attractive."

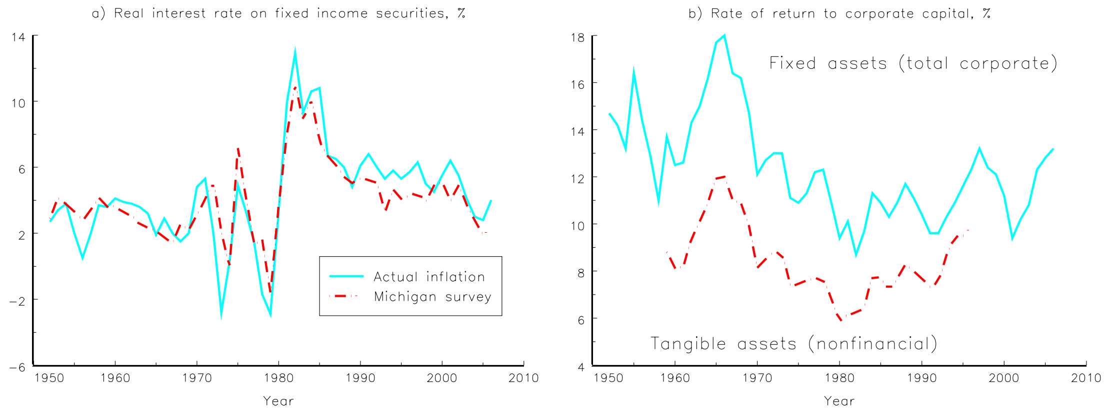

Figure 2: The real interest rate, U.S. 1952–2006

Notes: a) The real interest rate is calculated as BAA corporate bond yield minus expected inflation, where expected inflation is either actual one-year ahead inflation (solid line), or the inflation forecast from Michigan Consumer Attitudes survey (dashed line). b) The rate of return is before tax corporate profits divided by fixed assets (solid line), or tangible assets (dashed line). All interest rates are annualized and percent.
Sources: a) Moody's BAA corporate bond yield from Board of Governors of the Federal Reserve System, https://research.stlouisfed.org/fred2/, CPI (for all urban consumers and all items) inflation data are from Bureau of Labor Statistics (series ID CPIAUCNS), and Michigan Consumer Attitudes Survey. b) Poterba (1998, Table 1, column 1), and author's calculations based on BEA data as explained in the Data Appendix.

particular, I follow their lead in constructing empirical variables that match their theoretical counterparts as closely as possible. The most distinctive feature of the analysis is the quantitative methodology, which is discussed in section 3.2. I now briefly describe the baseline model without explicit modeling of housing. In the quantitative analysis, I extend the baseline model to include the production of home goods by means of housing as capital stock. The technical appendix A contains a more complete treatment of the baseline model, as well as the (extended) model with housing.

3.1 The basic environment

Households.—There is a representative household whose size N grows at a gross rate $n_t = N_t/N_{t-1}$. (N should be thought of as the civilian non-institutional population ages 16 and over.) Total market and non-market time available to each member of the household is normalized to one. Each member of this household derives utility from consuming market goods, c_{mt}, and disutility from working in the market, h_{mt}. The underlying preferences are represented by an intertemporal utility function

$$\sum_{t=0}^{T} \beta^t N_t u(C_t, 1 - h_t), \tag{1}$$

where $0 < \beta < 1$ is the discount factor. The instantaneous utility function is isoelastic,

$$u(C_t, 1 - h_t) = \frac{[C_t(1 - h_t)^{\omega}]^{1-\sigma}}{1 - \sigma}, \quad \sigma \neq 1, \tag{2}$$

where $\sigma > 0$ is the inverse of the elasticity of inter-temporal substitution, and $\omega > 0$ is the weight on leisure.[11]

The market capital stock evolves according to

$$n_{t+1}k_{\mathrm{m}t+1} = (1 - \delta_{\mathrm{m}t})k_{\mathrm{m}t} + x_{\mathrm{m}t}, \tag{3}$$

where the depreciation rate is denoted by δ_{m}, and investment level is x_{m}.

The rental rate on market capital is r, and the wage rate is w per unit of market hours. Both are determined in competitive factor markets. Capital income is taxed at $\tau_{\mathrm{k}t}$.[12] There are also transfers (or lump-sum taxes) from the government τ_t. The per capita household budget constraint is

$$c_{\mathrm{m}t} + x_{\mathrm{m}t} = w_t h_{\mathrm{m}t} + (1 - \tau_{\mathrm{k}t})r_t k_{\mathrm{m}t} + \tau_t. \tag{4}$$

Firms.— Firms operating in competitive product markets use a constant-returns-to-scale production function

$$y_{\mathrm{m}t} = k_{\mathrm{m}t}^{\alpha} \left(A_{\mathrm{m}t} h_{\mathrm{m}t}\right)^{1-\alpha}, \tag{5}$$

where $A_{\mathrm{m}t}$ is the exogenous labor-augmenting technology in the market sector. The parameter $0 < \alpha < 1$ is the elasticity of output with respect to capital, and it is time invariant.

Technological change.— Exogenous rates of labor-augmenting technological change are given by

$$g_{\mathrm{m}t} = \frac{A_{\mathrm{m}t}}{A_{\mathrm{m}t-1}}. \tag{6}$$

Government sector.— The government expenditures are equal to revenues in each period:

$$\tau_{\mathrm{k}t} r_t k_{\mathrm{m}t} = g_t + \tau_t, \tag{7}$$

where g is government consumption expenditures per person.

Equilibrium and the steady-state solution.— The equilibrium in this model requires that goods and labor markets clear simultaneously (again see the technical appendix). The model has a unique steady-state solution.

[11] When $\sigma = 1$, the utility function is $u(C_t, 1 - h_t) = \ln C_t + \omega \ln(1 - h_t)$.

[12] The tax rate on capital income is calculated as the ratio of capital income tax to before-tax profits plus capital consumption allowance. Thus, after-tax real interest rate is $(1 - \tau_{\mathrm{k}t})f'(k_{\mathrm{m}}) - \delta_{\mathrm{m}}$. Below I also allow for taxes on labor income.

3.2 Quantitative methodology

I solve the model numerically. The quantitative methodology takes as data initial conditions for endogenous state variables, parameter values, and exogenous variables, and then solves for optimal values of endogenous variables.

Specifically, the solution uses the following data: parameter values for β, α, ω and σ, which are either calibrated by matching "calibration targets" based on steady-state values of endogenous variables (such as β), or measured outside the model from actual data (such as α); exogenous variables $g_{\mathrm{m}t}, n_t, \delta_{\mathrm{m}t}, g_t$, and $\tau_{\mathrm{k}t}$, which are set to their empirical counterparts from actual data, and the initial value of the endogenous state variable $k_{\mathrm{m}t}$. I have 55 annual observations (from 1952 to 2006) on the exogenous variables. The data appendix B discusses the sources for parameter values and the estimated variables. The algorithm then solves for equilibrium values of $k_{\mathrm{m}t}, c_{\mathrm{m}t}$, and $h_{\mathrm{m}t}$, for $t = 0, \dots, T$.[13] Since this methodology uses the *actual* realizations of exogenous variables, I am able to directly compare the model-based endogenous variables with the actual data.[14]

4 The simulation results

How successful is this baseline model in matching the consumption–income ratio in the U.S.? Figure 3 panel (a) shows the simulation results for the basic model and compares them with the actual data—the ratio of *market* consumption to *market* income, whereby both measures exclude gross housing value added. The figure delivers three main messages. First, over much of the period, the model-based consumption–income ratio is significantly higher than the actual ratio in the data. The gap is especially significant at the beginning of the sample period (1952–19170).

Second, the model-based series are significantly more volatile than the data. This "excess smoothness" of the consumption–income ratio in actual data may be due to a variety of reasons. One possibility, which I pursue later on, is that the growth rates of labor-augmenting productivity index used in these simulations do not necessarily capture the measure of long-run

[13]The numerical solution is a forward-iteration algorithm as described in the technical appendix A.2. I determine the steady-state solution of the model using the sample averages of $g_{\mathrm{m}}, n, \delta_{\mathrm{m}}, g$, and τ_{k} as their steady-state counterparts. In order to solve $k_{\mathrm{m}t}, c_{\mathrm{m}t}$, and $h_{\mathrm{m}t}$, I set $T > 55$ to an arbitrarily large number. I check whether the endogenous variables converge sufficiently to their theoretical steady states. I verify this numerically by increasing the value of T. In practice, $T = 100$, corresponding to 2050 in calendar time, is sufficient to attain the convergence criterion.

[14]Previous studies that employ a similar quantitative methodology are Chen, İmrohoroğlu, and İmrohoroğlu (2007), and Braun, Ikeda, and Joines (2006), both for Japan, and Cooley and Ohanian (1997), for the U.K.

Figure 3: Simulation results for the baseline model

Notes: Consumption is personal consumption expenditures minus gross housing value added. Income is GDP minus gross housing value added. See Appendix B for data sources and parameter values underlying the model-based series. Model-based series are computed using a forward-iteration algorithm as explained in Appendix A.2. Out-of-sample values of $g_{\mathrm{m}}, n, \delta_{\mathrm{m}}$, and τ_{k} are set equal to their sample (1952–2006) averages. The steady-state value of h_{m} is also set equal to its sample average.

productivity growth rate on which the households and firms might have based their decisions.

Third, and the above mentioned shortcomings notwithstanding, the model-based consumption–income series track the data remarkably well. For instance, the model-based series parallel the fall in the consumption–income ratio in the 1960s, and the dramatic increase in the data since the early 1980s. Moreover, the model anticipates the decline in the consumption-to-income ratio that started in the data after 2004, although the model-based series decline at a much faster rate than it is apparent in the data.[15]

Figure 3 panel (b) shows the model-based and actual pre-tax rate of return to capital. The model-based series track the data well. While these 'visual checks' does not constitute a formal test, the trends in Figures 3 make a strong case for the significance of medium- to long-run swings in the productivity growth rate as the key determinant of the joint behavior of the consumption–income ratio and the rate of return to capital. At a more fundamental level, the results suggest that the key propagation mechanisms underlying the model, namely capital accumulation and elasticity of intertemporal substitution, have had an economically significant impact on the U.S. saving rate, and I discuss these channels next.

[15]In interpreting these results two additional issues should be kept in mind: (i) the model-based series are highly sensitive to the initial values of the capital–output ratio, and (ii) there may still remain a conceptual discrepancy between the model-based consumption and income series and the data published in the NIPA.

4.1 Productivity growth and the elasticity of intertemporal substitution

In this section, I take stock and present an economic explanation of the medium-run swings in the consumption–income ratio. In the model, this ratio is dictated by two structural factors: the medium-run growth rate of the labor augmenting technological progress and the elasticity of intertemporal substitution. The former determines the intertemporal tradeoffs, as manifested in the rate of return to capital, and the latter determines the degree to which the saving rate will respond to these tradeoffs.

It is particularly informative to study these channels through the lenses of productivity growth episodes. The vast literature on the measurement of productivity in the U.S. has pointed to several distinct episodes since the 1950s: 1952–1973 (*Golden Age* of productivity growth), 1973–1995 (productivity *slowdown*), and 1995–2006 (productivity *resurgence*).[16] Consider first the period from 1952 to 1973. During the Golden Age, the productivity growth rate was relatively high. In the model, high (but transitory) productivity growth stimulates investment, and thus leads to a relatively low consumption–income ratio. The corresponding combination of low consumption–income ratio and high rate of return to capital in actual data is evident in Figure 3.

Against this backdrop of an incentive to save, however, there were countervailing incentives to reduce the saving rate during the Golden Age (1952–1973). Note that the consumption–income ratio declines, albeit mildly, during the 1960s. There is a corresponding, but admittedly more pronounced, decline in the model-based series. This is in part due to the fact that throughout the 1950s, consumption was possibly below its steady-state level, still recovering from war time measures. The intertemporal substitution motive anticipates this relatively *higher* consumption–income ratio, at least in the first decade of the sample.[17]

The arrival of productivity slowdown from 1973 to 1995 reverses the economic incentives to save and invest: marginal product of capital declines, and investment falls. Despite relatively low income growth, however, the intertemporal substitution motive leads to sustained consumption growth. With this emerges a period of rising consumption–income ratio. The model thus tracks the rise in the actual consumption–income ratio and the fall in the rate of return to capital in Figure 3.

Productivity resurgence after 1995 initiates a new wave of rising consumption–income ratio. In the model, higher productivity growth leads to higher rates of investment, and a gradual rise

[16] See Gordon (2004), and Joregenson, Hu, and Stiroh (2008), and the references cited therein.

[17] Put differently, a higher elasticity of intertemporal substitution would have lead to a reduction in the consumption–income ratio at the beginning of the sample period.

in the consumption–income ratio—"gradual," because the growth rate of productivity has not been uniform from 1995 to 2006, and, on average, has not been as vigorous as the Golden Age. It is, thus, notable that the model-based series do not capture the consumption boom from 1995 to 2000.

4.2 Future of the consumption–income ratio

What is the baseline model's forecast about the future of the consumption–income ratio in the U.S.? The answer to this question depends on history for the simple reason that contemporary capital stock is the initial condition for this thought experiment. At the same time, and as it will become clear, the factors that determine the future of the consumption-income ratio have an impact on our interpretation of the past. To chart the future of the U.S. saving rate, I modify the basic setup in two dimensions: I use forecasts of (i) population, and (ii) labor-augmenting productivity factor growth rates from 2007 to 2050. Population forecasts span from 2006 to 2050, and are from U.S. Census Bureau (2004). For productivity forecasts, following Jorgenson et al. (2008), I consider three scenarios for labor-augmenting productivity factor growth rate, g_{m}, based on their total factor productivity growth rate scenarios: In the "pessimistic" case $g_{\mathrm{m}} = 1.0070$, in the "baseline" case it is 1.0113, and in the "optimistic" case it is 1.0132.[18]

Figure 4 shows the forecasts of consumption–income ratio in the U.S. under alternative productivity growth scenarios. While there is considerable uncertainty regarding the precise value of this ratio, all the scenarios point to a high (about 67%) and stable consumption–income ratio in the future. While this model-based forecast is above the average consumption–income ratio from 1952 to 1980, it is still below that of the actual consumption–income ratio as of 2007.

Figure 4 also raises two important issues. First, it demonstrates the material impact of productivity forecasts on the entire path of the consumption–income ratio. Indeed, different forecasts about productivity growth imply different paths for the consumption–income ratio—even though *in-sample* (1952–2006) productivity growth rates are identical under all three scenarios. In other words, statements about a "low" or "high" saving rate are inseparable from forecasts of *future* productivity growth paths: An "optimistic" forecast of productivity growth rate tends to imply on average a lower consumption–income ratio, both in and out of sample.

Second, a related issue that emerges from Figure 4 is the possibility of using real-time forecasts of productivity growth throughout the entire sample period, and not just at the end of the sample period. This issue is particularly relevant if surprise shifts in the medium-run productivity

[18] Their forecasts span from 2007 to 2016, but I use them as indicative of long-term forecasts until 2050.

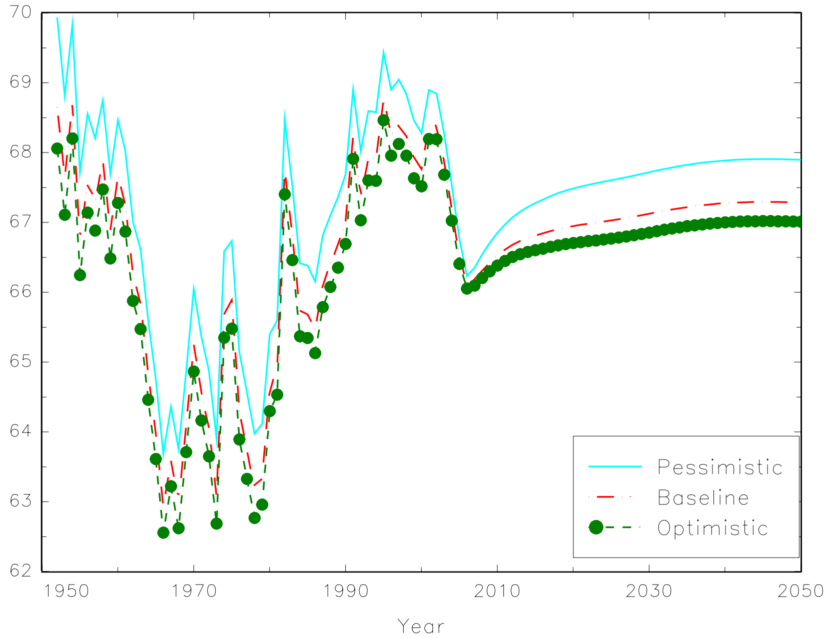

Figure 4: Forecasts of consumption–income ratio

Notes: Consumption is personal consumption expenditures minus gross housing value added. Income is GDP minus gross housing value added. See Appendix B for data sources and parameter values underlying the model-based series. Model-based series are computed using a forward-iteration algorithm as explained in Appendix A.2. The steady-state value of $h_{\rm m}$ is set equal to its sample average. Out-of-sample values of $\delta_{\rm m}$, and $\tau_{\rm k}$ are set equal to their sample (1952–2006) averages, and $g_{\rm m}$, and n are forecasts by Jorgenson, Hu, and Stiroh (2008), and U.S. Bureau of Census (2004), respectively. "Pessimistic," "baseline" and "optimistic" correspond to Jorgenson et al.'s forecast of productivity growth under alternative scenarios.

growth rate lead to a wedge between actual and forecasted (or perceived) productivity growth series. In the next section, I provide evidence on the significance of these forecasts for the model-based consumption–income ratio.

4.3 Real-time forecasts of productivity growth

The estimates of productivity growth rate based on a specific economic model *and* information available to us today indicate several distinct episodes. For interpreting the data, however, it is important to ascertain whether agents in real time had similar perceptions and were able to anticipate these turning points in the long-run productivity growth rate between these episodes. In a recent study, Edge, Laucbach, and Williams (2007) argue that these turning points were, by and large, unanticipated and, more importantly, perceived as long-term shifts only after a significant time lag.[19]

The *real-time* long-run productivity growth forecasts may differ from the actual estimates of productivity growth based on the revised estimates published by statistical agencies or economists (*revised data*, for short) for a variety of reasons. One possibility is that *perceptions* of productivity growth might be different from those used by statistical agencies or those based on economic models. Or, information available to forecasters in real time may be noisier. Whatever the ultimate drivers of these differences, the productivity growth rates based on real-time and revised data are significantly different. I thus explore whether simulating the baseline model using real time, instead of revised productivity growth rates makes material difference for the consumption–income ratio, and use the real-time Kalman filter estimates of Edge et al. (2007).[20]

Figure 5 shows the simulation results using the real-time and revised total factor productivity growth data.[21] I introduce the forecasted values of long-run productivity growth into the analysis in two ways. The results based on "real-time" estimates use the *entire* time path of the Kalman filter estimates. The results based on "forecast" use the forecasted values of productivity growth rate up to a given year, assume that the latest available forecast represents the long-run productivity growth rate henceforth, and updates these forecasts dynamically.[22]

These simulation results in Figure 5 are significant in several ways. First, consider the ratio of consumption to income from 1952 to 2050 in panel (a). The real-time based consumption–income ratio tends to be less volatile than the revised series. This is due to the fact that, compared to the revised series, Kalman-filter estimates of productivity growth have much less high-frequency variability. Also, the real-time TFP data imply significantly lower consumption–income ratio compared to revised TFP data, and much of this is driven by the lagged recognition of the productivity resurgence in the 1990s by the forecast data. This, however, imparts a relatively larger capital–output ratio to the real-time TFP based series. Consequently, while both revised

[19] We may be faced with a similar swing at the time of this writing: productivity growth rate has slowed down in recent years, and, there is no consensus among scholars and professional forecasters regarding the viability of the recent productivity resurgence (the "optimistic" scenario in section 4.2).

[20] Council of Economic Advisors (CEA) also forecast long-run productivity growth. However, their published forecasts start in 1969 and are not available for a number of years. Edge et al.'s (2007) estimates start in 1965 and end in 2005, and are consistent with the CEA forecasts in those years for which there is data.

[21] The original real-time data correspond to forecasts of labor productivity growth rate. I converted these into forecasts of labor-augmenting productivity growth; see appendix B. Kalman-filter forecasts start in 1965, so for 1952–1965 I used the revised TFP data. The forecast for long-term productivity growth underlying the model-based series is the "baseline" TFP growth forecast of Jorgenson et al. (2008).

[22] The exercise here is exploratory. Without taking a firm stance on a model of learning or perceptions, there is no satisfactory way to handle the discrepancy between revised and real-time values of labor productivity.

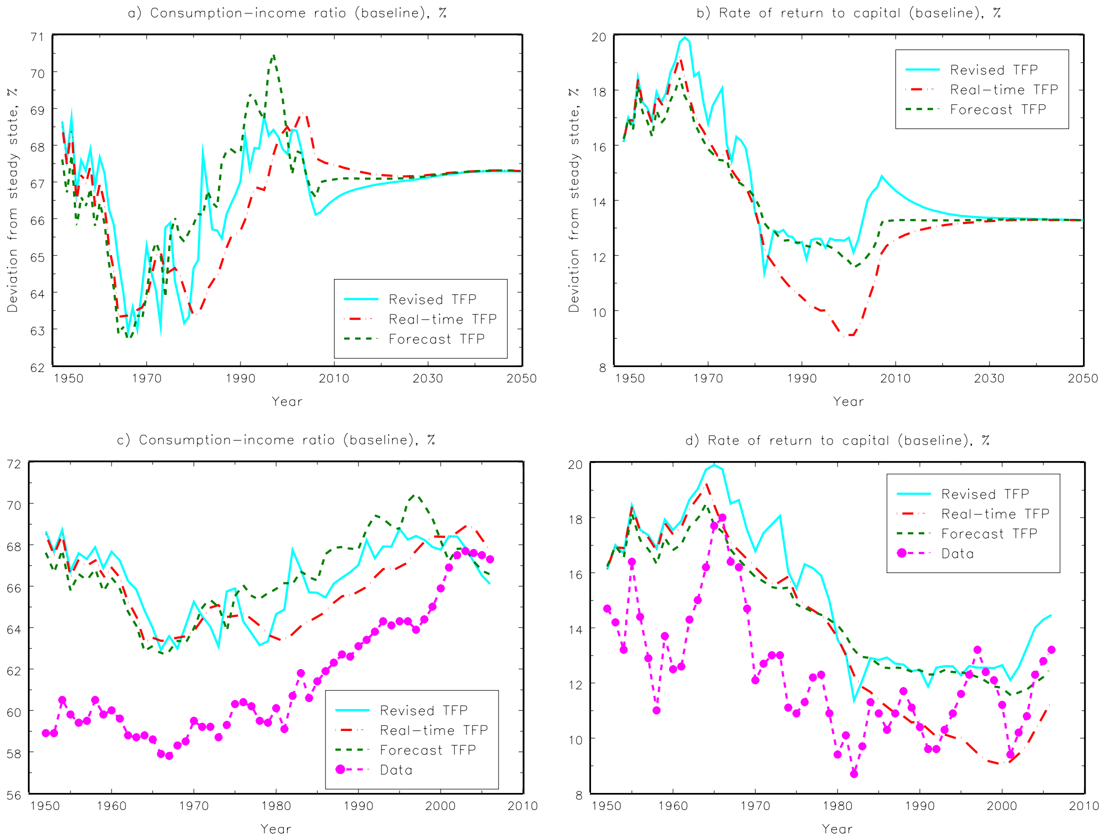

Figure 5: Simulation results with real-time and revised productivity data

Notes: Consumption is personal consumption expenditures minus gross housing value added. Income is GDP minus gross housing value added. See Appendix B for data sources and parameter values underlying the model-based series. Model-based series are computed using a forward-iteration algorithm as explained in Appendix A.2. Out-of-sample values of δ_{m}, and τ_{k} are set equal to their sample (1952–2006) averages, and g_{m}, and n are forecasts by Jorgenson, Hu, and Stiroh (2008), and U.S. Bureau of Census (2004), respectively. Steady-state value of h_{m} is set equal to its sample average. Real-time TFP forecasts are Kalman filter estimates in Edge et al. (2007) with optimal gain parameter 0.11.

and real-time data indicate a decline in the U.S. consumption–income ratio, the fall in the real-time based data is considerably more gradual.

Figure 5 panel (b) shows the pre-tax rate of return to capital using both revised and real-time TFP data. There are significant differences between the simulation results based on revised and real-time data, especially after the early 1980s. Based on the revised TFP data, the rate of return to capital is fairly stable. Based on the real-time data, however, it falls secularly from 1980 until the end of 1990. In both cases, the rate of return increases rapidly after 2000. In the case of real-time data, it approaches to its value along the balanced growth path from below, and in the case of revised data from above.

In Figure 5 panels (c) and (d), I repeat the same exercise, and this time compare the simulated consumption–income ratio and rate of return to capital series with the actual data (1952–2006). In terms of congruity over time in both the levels and the broader trends in each of these variables, the real-time TFP based model performs better. There are, of course, several instances of significant gaps between the actual and the model-based series—the rate of return data in the 1990s being one of them. Overall, then, in the context of consumption-income ratio, the use of real-time forecasts of productivity growth help bring the model closer to data, and the results are suggestive of the economically significant impact of perceived shifts in long-run productivity growth on the saving rate.[23]

4.4 Housing

In recent years, housing wealth has attracted considerable attention in the context of the consumption boom in the U.S.—and elsewhere (Altissimo et al., 2005). The baseline model excludes this non-negligible fraction of total wealth. One way to incorporate housing into the model, and the one I pursue, is to embed housing within a non-market production function framework. In this approach housing stock is exclusively used, in conjunction with hours worked at home, to produce a non-market good.

This extension leads to a richer and a more parameterized model. (See appendix A for details.) Figure 6 focuses on the consumption–income ratio and compares the model-based series from the models with and without housing with the actual data. Modeling choices about residential housing leads to a significant improvement in the performance of the model relative to the baseline. The model-based consumption–income series track the actual data very closely, especially from about 1970 until 2000. This suggests that modeling home production in general, and housing in particular has an economically significant impact on the interpretation of the saving rate in the U.S.[24]

At the same time, the model with housing has two principle quantitative shortcomings. First, it yields a much higher consumption-income ratio at the beginning of the sample period—and much more so than the baseline model. Second, the model shows a marked decline in the consumption–income ratio, at least since 2000. Note that this is roughly the beginning of

[23] I also computed the model with labor income taxes, and not surprisingly the results are very similar to those reported above (see Figure 6 below).

[24] I also considered durable goods as an input to home production. The results, which are available from the author, suggest that accounting for durable goods does not have significant impact on the model-based consumption-to-income ratio.

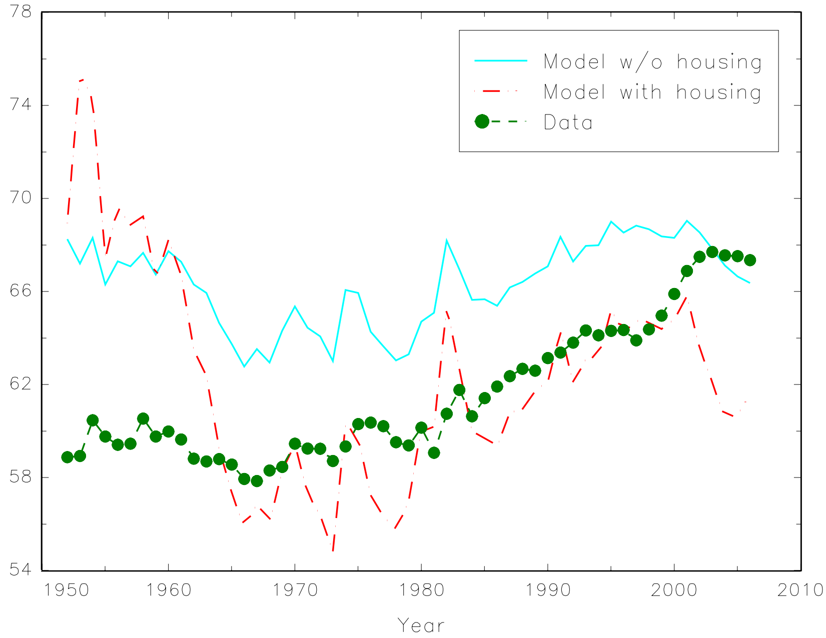

Figure 6: Consumption–income ratio with and without housing

Notes: Consumption is personal consumption expenditures minus gross housing value added. Income is GDP minus gross housing value added. See Appendix B for data sources and parameter values underlying the model-based series. Model-based series are computed using a forward-iteration algorithm as explained in Appendix A.2. Out-of-sample values of $\delta_{\mathrm{m}}, \tau_{\mathrm{k}}$ and τ_{ℓ} are set equal to their sample (1952–2006) averages, and g_{m}, and n are forecasts by Jorgenson, Hu, and Stiroh (2008), and U.S. Bureau of Census (2004), respectively. Steady-state value of h_{m} is set equal to its sample average.

the run-up in housing prices in the United States, as well as relatively large current account deficits.[25] I conjecture that the model cannot account for these massive capital gains in the housing sector, and growing global imbalances, and thus indicates a sharper reduction in the consumption-to-income ratio, in tandem with a slowdown in productivity growth.[26]

[25]In this context, the closed economy model performs very well before the rise in U.S. net foreign asset liabilities. In a set of complementary papers, Ghironi et al. (2008), and Chen et al. (2008) analyze the contribution of differential productivity growth in the United States and abroad to changes in international investment position of the United States. They do not however consider home production, and the differences between forecasted versus actual productivity growth rates.

[26]One key assumption underlying the quantitative results in this section is that labor augmenting technology in the market and in the non-market sectors are identical. Non-market sector output is not measured, so there is *no* established procedure to estimate the growth rate of productivity in this sector. Given the uncertainty surrounding the non-market productivity growth figures, these results should be viewed as suggestive. For this reason, I do not compute the model-based consumption-income ratio using real-time forecasts.

5 Concluding remarks

The general equilibrium analysis I presented in this paper relies on a highly compact (and stylized) modeling approach. It emphasizes an empirical strategy that maps model-based variables to their empirical counterparts. In fact, throughout much of the analysis, I focused on relatively narrow concepts of market consumption and income, and particularly on those to which the model can speak. The baseline model matches the broader medium-run variations in the consumption–income ratio and rate of return to capital in the United States quite well, but it is still not completely satisfactory. The extended model with housing leads to a better match between the actual and model-based series. However, this model also fails to account for the persistent and relatively higher consumption-income ratio at the end of the period of analysis.

There is another message of this analysis: the economy-wide productivity growth rate in the United States exhibited substantial medium-run swings since the post war, and these had economically significant impact on the observed saving rate. Building on the recent research, I argued that real time forecasts of medium-run productivity growth provide a better understanding of the actual data. In this context, building models with learning as a way to study aggregate saving rates strike me as a promising avenue to pursue, and is left for future research.

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Appendix

A Derivations

In this appendix, section A.1 presents the general model with non-market production. Section A.2 discusses the solution algorithm. Section A.3.1 presents in detail the baseline model discussed in sections 3. Finally, section A.3.2 presents the extended model with housing discussed in section 4.4.

A.1 The model economy

Households.— There is a representative household whose size N grows at a gross rate $n_t = N_t/N_{t-1}$. (N should be thought of as the civilian non-institutional population ages 16 and over.) Total market and non-market time available to each member of the household is normalized to one. Each member of this household derives utility from consuming market goods, $c_{\mathrm{m}t}$, and home goods $c_{\mathrm{h}t}$, and disutility from working either in the market, $h_{\mathrm{m}t}$, or at home, $h_{\mathrm{h}t}$. The underlying preferences are represented by

$$\sum_{t=0}^{T} \beta^t N_t U \left(\mathcal{C}(c_{\mathrm{m}t}, c_{\mathrm{h}t}), h_{\mathrm{m}t}, h_{\mathrm{h}t}\right), \tag{A.1}$$

where $0 < \beta < 1$ is the discount factor, and $\mathcal{C}$ is a homogenous of degree one consumption aggregator. The government provides services, g_t, but the level of these services is not a decision variable.

Home production uses housing capital, $K_{\mathrm{h}t}$, as well as home hours according to

$$c_{\mathrm{h}t} = H(k_{\mathrm{h}t}, A_{\mathrm{h}t} h_{\mathrm{h}t}), \tag{A.2}$$

where $A_{\mathrm{h}t}$ is the exogenous labor-augmenting technology in the home sector. H is homogenous of degree one, so $k_{\mathrm{h}t} = K_{\mathrm{h}t}/N_t$ is per capita durable goods stocks.

There is also capital stock used in market production, $k_{\mathrm{m}t}$. The home and market capital stocks evolve according to

$$n_{t+1} k_{\mathrm{h}t+1} = (1 - \delta_{\mathrm{h}t}) k_t + x_{\mathrm{d}t}, \tag{A. 3a}$$

$$n_{t+1} k_{\mathrm{m}t+1} = (1 - \delta_{\mathrm{m}t}) k_t + x_{\mathrm{m}t}. \tag{A. 3b}$$

where the depreciation rates are denoted by δ, and investment levels are by x.

The rental rate on market capital is r, and the wage rate is w per unit of market hours. Both are determined in competitive factor markets. These earnings are taxed at $\tau_{\mathrm{k}t}$ and $\tau_{\ell t}$, respectively.[27] There are also transfers (or lump-sum taxes) from the government τ_t. The per capita household budget constraint is

$$c_{\mathrm{m}t} + x_{\mathrm{h}t} + x_{\mathrm{m}t} = (1 - \tau_{\ell t}) w_t h_{\mathrm{m}t} + (1 - \tau_{\mathrm{k}t}) r_t k_{\mathrm{m}t} + \tau_t. \tag{A. 4}$$

[27] The tax rate on capital is calculated as the ratio of capital income tax to before tax profits plus capital consumption allowance. Thus, after-tax real interest rate is $(1 - \tau_{\mathrm{k}t}) f'(k_{\mathrm{m}}) - \delta_{\mathrm{m}}$.

Firms.— Firms operating in competitive product markets use a constant-returns-to-scale production function F

$$y_{\mathrm{m}t} = F(k_{\mathrm{m}t}, A_{\mathrm{m}t}h_{\mathrm{m}t}), \tag{A. 5}$$

where $A_{\mathrm{m}t}$ is the exogenous labor-augmenting technology in the market sector.

Technological change.— Exogenous rates of technological change are given by

$$g_{\mathrm{h}t} = \frac{A_{\mathrm{h}t}}{A_{\mathrm{h}t-1}}, \tag{A. 6a}$$

$$g_{\mathrm{m}t} = \frac{A_{\mathrm{m}t}}{A_{\mathrm{m}t-1}}. \tag{A. 6b}$$

Equilibrium conditions.— The market-clearing and first-order conditions are

$$h_{\mathrm{h}t} + h_{\mathrm{m}t} + h_{\ell\, t} = 1, \tag{A. 7a}$$

$$c_{\mathrm{h}t} = H(k_{\mathrm{h}t}, k_{\mathrm{h}t}, A_{\mathrm{h}t}h_{\mathrm{h}t}), \tag{A. 7b}$$

$$c_{\mathrm{m}t} + x_{\mathrm{h}t} + x_{\mathrm{h}t} + x_{\mathrm{m}t} = (1 - \tau_{\ell\, t})w_t h_{\mathrm{m}t} + (1 - \tau_{\mathrm{k}t})r_t k_{\mathrm{m}t} + \tau_t, \tag{A. 7c}$$

$$(1 - \tau_{\ell\, t})\, F_{\mathrm{h}}(t)\, U_{\mathrm{c}}(t)\, \mathcal{C}_{\mathrm{m}}(t) + U_{\mathrm{h}_m}(t) = 0, \tag{A. 7d}$$

$$H_{\mathrm{h}}(t)\, U_{\mathrm{c}}(t)\, \mathcal{C}_{\mathrm{h}}(t) + U_{\mathrm{h}_h}(t) = 0, \tag{A. 7e}$$

$$U_{\mathrm{c}}(t)\, \mathcal{C}_{\mathrm{m}}(t) = \beta\, U_{\mathrm{c}}(t+1)\, \mathcal{C}_{\mathrm{m}}(t+1)\, [(1 - \tau_{\mathrm{k}t+1})F_{\mathrm{k}}(t+1) + 1 - \delta_{\mathrm{m}t+1}], \tag{A. 7f}$$

$$U_{\mathrm{c}}(t)\, \mathcal{C}_{\mathrm{m}}(t) = \beta\, U_{\mathrm{c}}(t+1)\, \left[\mathcal{C}_{\mathrm{h}}(t+1) H_{\mathrm{k}_d}(t+1) + (1 - \delta_{\mathrm{h}t+1})\mathcal{C}_{\mathrm{m}}(t+1)\right]. \tag{A. 7g}$$

Total time is normalized to 1, and h_ℓ is leisure. The first three conditions state the equality of supply and demand in labor and goods markets, followed by two intra-temporal optimality conditions for consumption–leisure choice. The last two conditions are the intra-temporal optimality conditions for consumption growth based on real rates of return to two types of physical capital: in each of these equations, the left-hand side is the utility loss of additional saving in time period t, and the right-hand side is the utility gain in period $t+1$ arising from investing that additional saving in any one type of capital. In the above expressions, subscripts after $U, \mathcal{C}, F$, and H denote the partial derivatives. These, together with the capital accumulation equations and the government budget constraint

$$\tau_{\mathrm{k}t} r_t k_{\mathrm{m}t} + \tau_{\ell t} w_t h_{\mathrm{m}t} = g_t + \tau_t, \tag{A. 8}$$

where g is government consumption expenditures per person, determine the equilibrium in this model. In the above set of equations

1. the control and endogenous state variables (unknowns) are $c_{\mathrm{m}t}, c_{\mathrm{h}t},\ x_{\mathrm{h}t}, x_{\mathrm{m}t}, h_{\mathrm{h}t}, h_{\mathrm{m}t}$, $k_{\mathrm{h}t+1}$, and $k_{\mathrm{m}t+1}$,

2. the exogenous variables are $n_t, \delta_{\mathrm{h}t}, \delta_{\mathrm{m}t}, \tau_{\ell\, t}, \tau_{\mathrm{k}t}, g_{\mathrm{h}t}$, and $g_{\mathrm{m}t}$,and

3. the initial conditions are N_0, $k_{\mathrm{h}0}$, $k_{\mathrm{m}0}$, $A_{\mathrm{h}0}$ and $A_{\mathrm{m}0}$.

A.2 The computational algorithm

I use a forward-iteration algorithm to numerically solve several parameterized versions of this problem. Heer and Maußner (2005) provide a clear exposition of this approach.

A.3 Parametrization

I first define the transformed variables

$$\tilde{c}_{\mathrm{m}t} \equiv \frac{c_{\mathrm{m}t}}{A_{\mathrm{m}t}}, \quad \tilde{k}_{\mathrm{m}t} \equiv \frac{k_{\mathrm{m}t}}{A_{\mathrm{m}t}}, \quad \tilde{g}_{\mathrm{m}t} \equiv \frac{g_t}{A_{\mathrm{m}t}}. \tag{A. 9}$$

A.3.1 The baseline model

The baseline model incorporates the government spending, taxation decisions, and variable labor supply, but excludes home production.

The instantaneous utility function is

$$U(c_{\mathrm{m}t}, h_{\mathrm{m}t}) = \frac{\left[c_{\mathrm{m}t}(1-h_{\mathrm{m}t})^{\omega}\right]^{1-\sigma}}{1-\sigma}, \quad \sigma \neq 1, \tag{A. 10}$$

where $\sigma > 0$ is the inverse of the elasticity of inter-temporal substitution. When $\sigma = 1$, the utility function is

$$U(c_{\mathrm{m}t}, h_{\mathrm{m}t}) = \ln c_{\mathrm{m}t} + \omega \ln(1-h_{\mathrm{m}t}). \tag{A. 11}$$

The (market) production function is Cobb-Douglas

$$F(k_{\mathrm{m}t}, A_{\mathrm{m}t}h_{\mathrm{m}t}) = k_{\mathrm{m}t}^{\alpha}\left(A_{\mathrm{m}t}h_{\mathrm{m}t}\right)^{1-\alpha}, \quad 0 < \alpha < 1. \tag{A. 12}$$

The set of first-order difference equations corresponding to the optimal transformed variables are

$$\left(\frac{\tilde{c}_{\mathrm{m}t+1}}{\tilde{c}_{\mathrm{m}t}}\right)^{\sigma}\left(\frac{1-h_{\mathrm{m}t+1}}{1-h_{\mathrm{m}t}}\right)^{\omega(\sigma-1)} = \frac{\beta}{g_{\mathrm{m}t+1}^{\sigma}}\left[(1-\tau_{\mathrm{k}t+1})\,\alpha\tilde{k}_{\mathrm{m}t+1}^{\alpha-1}h_{\mathrm{m}t+1}^{1-\alpha} + 1 - \delta_{\mathrm{m}t+1}\right], \tag{A. 13a}$$

$$n_{t+1}\, g_{\mathrm{m}t+1}\, \tilde{k}_{\mathrm{m}t+1} = \tilde{k}_{\mathrm{m}t}^{\alpha}h_{\mathrm{m}t}^{1-\alpha} - \tilde{g}_t + (1-\delta_{\mathrm{m}t})\tilde{k}_{\mathrm{m}t} - \tilde{c}_{\mathrm{m}t}, \tag{A. 13b}$$

where the market-hours worked satisfies the intra-temporal optimality condition

$$\frac{\omega\,\tilde{c}_{\mathrm{m}t}}{1-h_{\mathrm{m}t}} = (1-\alpha)\tilde{k}_{\mathrm{m}t}^{\alpha}h_{\mathrm{m}t}^{-\alpha}. \tag{A. 14}$$

The steady-state values of four endogenous variables $h_{\mathrm{m}}^*, \tilde{y}_{\mathrm{m}}^*, \tilde{k}_{\mathrm{m}}$, and $\tilde{c}_{\mathrm{m}}$ satisfy

$$1 = \frac{\beta}{g_{\mathrm{m}}^{\sigma}}\left[(1-\delta_{\mathrm{m}}) + (1-\tau_{\mathrm{k}})\alpha\left(\frac{\tilde{y}_{\mathrm{m}}^*}{\tilde{k}_{\mathrm{m}}^*}\right)\right], \tag{A. 15a}$$

$$\frac{\tilde{y}_{\mathrm{m}}^*}{\tilde{k}_{\mathrm{m}}^*} = \frac{\tilde{c}_{\mathrm{m}}^*}{\tilde{k}_{\mathrm{m}}^*} + \tilde{g} - (1-\delta_{\mathrm{m}} - n\, g_{\mathrm{m}}), \tag{A. 15b}$$

$$\frac{h_{\mathrm{m}}^*}{1-h_{\mathrm{m}}^*} = \left(\frac{1-\alpha}{\omega}\right)\left(\frac{\tilde{y}_{\mathrm{m}}^*}{\tilde{c}_{\mathrm{m}}^*}\right), \tag{A. 15c}$$

$$\frac{\tilde{y}_{\mathrm{m}}^*}{\tilde{k}_{\mathrm{m}}^*} = (h_{\mathrm{m}}^*)^{1-\alpha}\left(\tilde{k}_{\mathrm{m}}^*\right)^{\alpha-1}. \tag{A. 15d}$$

Following the standard calibration methodology, I set the steady-state values of h^*_{m}, and $\tilde{y}^*_{\mathrm{m}}/\tilde{k}_{\mathrm{m}}$ as "targets," and work with the implied values of β and ω, thereby maintaining four equations in four unknowns.

The set of nonlinear equations, whose (unique) solution is also the solution to the original problem, are

for $t = 0, \ldots, t' - 2$,

$$0 = \left[\frac{\tilde{k}^{\alpha}_{\mathrm{m}t+1} h^{1-\alpha}_{\mathrm{m}t+1} - \tilde{g}_{t+1} + (1-\delta_{\mathrm{m}t+1})\tilde{k}_{\mathrm{m}t+1} - n_{t+2}\, g_{\mathrm{m}t+2}\, \tilde{k}_{\mathrm{m}t+2}}{\tilde{k}^{\alpha}_{\mathrm{m}t} h^{1-\alpha}_{\mathrm{m}t} - \tilde{g}_{t} + (1-\delta_{\mathrm{m}t})\tilde{k}_{\mathrm{m}t} - n_{t+1}\, g_{\mathrm{m}t+1}\, \tilde{k}_{\mathrm{m}t+1}}\right]^{-\sigma} \left(\frac{1-h_{\mathrm{m}t+1}}{1-h_{\mathrm{m}t}}\right)^{\omega(\sigma-1)} - \frac{\beta}{g^{\sigma}_{\mathrm{m}t+1}} \left[(1-\tau_{\mathrm{k}t+1})\,\alpha\, \tilde{k}^{\alpha-1}_{\mathrm{m}t+1} h^{1-\alpha}_{\mathrm{m}t+1} + 1 - \delta_{\mathrm{m}t+1}\right], \tag{A. 16a}$$

$$0 = \tilde{k}^{\alpha}_{\mathrm{m}t} h^{1-\alpha}_{\mathrm{m}t} - \tilde{g}_{t} + (1-\delta_{\mathrm{m}t})\tilde{k}_{\mathrm{m}t} - n_{t+1}\, g_{\mathrm{m}t+1}\, \tilde{k}_{\mathrm{m}t+1} - \frac{1-\alpha}{\omega}(1-h_{\mathrm{m}t})\, \tilde{k}^{\alpha}_{\mathrm{m}t} h^{-\alpha}_{\mathrm{m}t}, \tag{A. 16b}$$

for $t = t' - 1$,

$$0 = \left[\frac{\tilde{k}^{\alpha}_{\mathrm{m}t+1} h^{1-\alpha}_{\mathrm{m}t+1} - \tilde{g}_{t+1} + (1-\delta_{\mathrm{m}t+1})\tilde{k}_{\mathrm{m}t+1} - n\, g_{\mathrm{m}}\, \tilde{k}_{\mathrm{m}t+2}}{\tilde{k}^{\alpha}_{\mathrm{m}t} h^{1-\alpha}_{\mathrm{m}t} - \tilde{g}_{t} + (1-\delta_{\mathrm{m}t})\tilde{k}_{\mathrm{m}t} - n_{t+1}\, g_{\mathrm{m}t+1}\, \tilde{k}_{\mathrm{m}t+1}}\right]^{-\sigma} \left(\frac{1-h_{\mathrm{m}t+1}}{1-h_{\mathrm{m}t}}\right)^{\omega(\sigma-1)} - \frac{\beta}{g^{\sigma}_{\mathrm{m}t+1}} \left[(1-\tau_{\mathrm{k}t+1})\,\alpha\, \tilde{k}^{\alpha-1}_{\mathrm{m}t+1} h^{1-\alpha}_{\mathrm{m}t+1} + 1 - \delta_{\mathrm{m}t+1}\right], \tag{A. 16c}$$

$$0 = \tilde{k}^{\alpha}_{\mathrm{m}t} h^{1-\alpha}_{\mathrm{m}t} - \tilde{g}_{t} + (1-\delta_{\mathrm{m}t})\tilde{k}_{\mathrm{m}t} - n_{t+1}\, g_{\mathrm{m}t+1}\, \tilde{k}_{\mathrm{m}t+1} - \frac{1-\alpha}{\omega}(1-h_{\mathrm{m}t})\, \tilde{k}^{\alpha}_{\mathrm{m}t} h^{-\alpha}_{\mathrm{m}t}, \tag{A. 16d}$$

for $t = t'$,

$$0 = \left[\frac{\tilde{k}^{\alpha}_{\mathrm{m}t+1} h^{1-\alpha}_{\mathrm{m}t+1} - \tilde{g} + (1-\delta_{\mathrm{m}})\tilde{k}_{\mathrm{m}t+1} - n\, g_{\mathrm{m}}\, \tilde{k}_{\mathrm{m}t+2}}{\tilde{k}^{\alpha}_{\mathrm{m}t} h^{1-\alpha}_{\mathrm{m}t} - \tilde{g}_{t} + (1-\delta_{\mathrm{m}t})\tilde{k}_{\mathrm{m}t} - n\, g_{\mathrm{m}}\, \tilde{k}_{\mathrm{m}t+1}}\right]^{-\sigma} \left(\frac{1-h_{\mathrm{m}t+1}}{1-h_{\mathrm{m}t}}\right)^{\omega(\sigma-1)} - \frac{\beta}{g^{\sigma}_{\mathrm{m}}} \left[(1-\tau_{\mathrm{k}})\,\alpha\, \tilde{k}^{\alpha-1}_{\mathrm{m}t+1} h^{1-\alpha}_{\mathrm{m}t+1} + 1 - \delta_{\mathrm{m}}\right], \tag{A. 16e}$$

$$0 = \tilde{k}^{\alpha}_{\mathrm{m}t} h^{1-\alpha}_{\mathrm{m}t} - \tilde{g}_{t} + (1-\delta_{\mathrm{m}t})\tilde{k}_{\mathrm{m}t} - n\, g_{\mathrm{m}}\, \tilde{k}_{\mathrm{m}t+1} - \frac{1-\alpha}{\omega}(1-h_{\mathrm{m}t})\, \tilde{k}^{\alpha}_{\mathrm{m}t} h^{-\alpha}_{\mathrm{m}t}, \tag{A. 16f}$$

for $t = t' + 1, \ldots, T - 2$,

$$0 = \left[\frac{\tilde{k}^{\alpha}_{\mathrm{m}t+1} h^{1-\alpha}_{\mathrm{m}t+1} - \tilde{g} + (1-\delta_{\mathrm{m}})\tilde{k}_{\mathrm{m}t+1} - n\, g_{\mathrm{m}}\, \tilde{k}_{\mathrm{m}t+2}}{\tilde{k}^{\alpha}_{\mathrm{m}t} h^{1-\alpha}_{\mathrm{m}t} - \tilde{g} + (1-\delta_{\mathrm{m}})\tilde{k}_{\mathrm{m}t} - n\, g_{\mathrm{m}}\, \tilde{k}_{\mathrm{m}t+1}}\right]^{-\sigma} \left(\frac{1-h_{\mathrm{m}t+1}}{1-h_{\mathrm{m}t}}\right)^{\omega(\sigma-1)} - \frac{\beta}{g^{\sigma}_{\mathrm{m}}} \left[(1-\tau_{\mathrm{k}})\,\alpha\, \tilde{k}^{\alpha-1}_{\mathrm{m}t+1} h^{1-\alpha}_{\mathrm{m}t+1} + 1 - \delta_{\mathrm{m}}\right], \tag{A. 16g}$$

$$0 = \tilde{k}^{\alpha}_{\mathrm{m}t} h^{1-\alpha}_{\mathrm{m}t} - \tilde{g} + (1-\delta_{\mathrm{m}})\tilde{k}_{\mathrm{m}t} - n\, g_{\mathrm{m}}\, \tilde{k}_{\mathrm{m}t+1} - \frac{1-\alpha}{\omega}(1-h_{\mathrm{m}t})\, \tilde{k}^{\alpha}_{\mathrm{m}t} h^{-\alpha}_{\mathrm{m}t}, \tag{A. 16h}$$

for $t = T - 1$,

$$0 = \tilde{k}^{\alpha}_{\mathrm{m}t} h^{1-\alpha}_{\mathrm{m}t} - \tilde{g}_{t} + (1-\delta_{\mathrm{m}t})\tilde{k}_{\mathrm{m}t} - n\, g_{\mathrm{m}}\, \tilde{k}^{*} - \frac{1-\alpha}{\omega}(1-h_{\mathrm{m}t})\, \tilde{k}^{\alpha}_{\mathrm{m}t} h^{-\alpha}_{\mathrm{m}t}, \tag{A. 16i}$$

with $\tilde{k}_{\mathrm{m}T} = \tilde{k}^*_{\mathrm{m}}$. Note that $\tilde{k}_{\mathrm{m}0}$, and $\{n_t, g_{\mathrm{m}t}, \delta_{\mathrm{m}t}, \tilde{g}_t\}_{t=0}^{t'}$ are data driven. I solve this nonlinear system of equations for $\left\{\tilde{k}_{\mathrm{m}t}, h_{\mathrm{m}t}\right\}_{t=1}^{T-1}$ numerically with a Newton-Raphson algorithm. And, I use equation (A. 13a) to solve for the sequence of per effective worker market consumption, $\{\tilde{c}_{\mathrm{m}t}\}_{t=0}^{T}$.

Experiment 1.— In this experiment, I modify the basic setup in two dimensions: I use forecasts of population and labor-augmenting productivity factor growth rates from 2007 to 2050. For productivity forecasts, following Jorgenson et al. (2008), I consider three scenarios for labor-augmenting productivity factor growth rate corresponding to their total factor productivity growth rate scenarios of 0.50, 0.81, and 0.95 (with annual percent growth rates): "Pessimistic" 0.70, "baseline," 1.13, and "optimistic" 1.32—though their forecasts span only until 2016.

Experiment 2.— In this experiment, I modify basic setup as in experiment 1, as well as by using the entire sample path of the real-time long-run productivity growth forecasts. To this end, I use the real-time Kalman filter estimates of Edge et al. (2007). Their estimates start in 1965 and end in 2005.

Experiment 3.— Here, I modify experiment 2 by using real-time long-run productivity growth forecasts in a dynamic fashion, whereby each productivity forecast is considered as a long-run forecast. These forecasts are updated with the intention of mimicking the behaviour of the economy by only using the information (for productivity) that was available to the agents in real time.

Experiment 4.— This experiment augments the model with direct taxes on labor, and is a straightforward extension of experiment 1.

A.3.2 The extended model with housing

I now include the optimal choice of non-market ("home") goods into the analysis, and confine my attention to housing as the only capital good in home production.

The parametrization of the model with housing stock only is as follows. The instantaneous utility function is

$$U(\mathcal{C}_t, h_{\mathrm{m}t}, h_{\mathrm{h}t}) = \frac{[\mathcal{C}_t(1 - h_{\mathrm{m}t} - h_{\mathrm{h}t})^{\omega}]^{1-\sigma}}{1-\sigma}, \quad \sigma \neq 1, \tag{A. 17}$$

where $\sigma > 0$ is the inverse of the elasticity of inter-temporal substitution. When $\sigma = 1$, the utility function is

$$U(\mathcal{C}_t, h_{\mathrm{m}t}, h_{\mathrm{h}t}) = \ln \mathcal{C}_t + \omega \ln(1 - h_{\mathrm{m}t} - h_{\mathrm{h}t}). \tag{A. 18}$$

The utility function for the combined home and market goods is

$$\mathcal{C}(c_{\mathrm{m}t}, c_{\mathrm{h}t}) = c_{\mathrm{m}t}^{\nu} c_{\mathrm{h}t}^{1-\nu}, \quad 0 < \nu < 1. \tag{A. 19}$$

The market sector production function is Cobb-Douglas,

$$F(k_{\mathrm{m}t}, A_{\mathrm{m}t}h_{\mathrm{m}t}) = k_{\mathrm{m}t}^{\alpha} \left(A_{\mathrm{m}t}h_{\mathrm{m}t}\right)^{1-\alpha}, \quad 0 < \alpha < 1. \tag{A. 20}$$

The home sector production function is Cobb-Douglas,

$$H(k_{\mathrm{h}t}, A_{\mathrm{h}t}h_{\mathrm{h}t}) = k_{\mathrm{h}t}^{\alpha_{\mathrm{h}}} \left(A_{\mathrm{h}t}h_{\mathrm{h}t}\right)^{1-\alpha_{\mathrm{h}}}, \quad 0 < \alpha_{\mathrm{h}} < 1. \tag{A. 21}$$

The following two first-order conditions characterize the optimal allocation of time between the home and market work:

$$\frac{\omega\,\tilde{c}_{\mathrm{m}t}}{\nu\,(1 - h_{\mathrm{m}t} - h_{\mathrm{h}t})} = (1-\tau_{\ell\,t})(1-\alpha)\tilde{k}_{\mathrm{m}t}^{\alpha} h_{\mathrm{m}t}^{-\alpha}, \tag{A. 22}$$

$$\frac{\omega\,\tilde{c}_{\mathrm{h}t}}{(1-\nu)(1 - h_{\mathrm{h}t})} = (1-\alpha_{\mathrm{h}})\tilde{k}_{\mathrm{h}t}^{\alpha_{\mathrm{h}}} h_{\mathrm{h}t}^{-\alpha_{\mathrm{h}}}. \tag{A. 23}$$

The following three first-order conditions characterize the optimal allocation of saving across market capital, and durable goods:

$$\begin{aligned}
&\left(\frac{\tilde{\mathcal{C}}_{t+1}}{\tilde{\mathcal{C}}_{t}}\right)^{\sigma-1} \left(\frac{\tilde{c}_{\mathrm{m}t+1}}{\tilde{c}_{\mathrm{m}t}}\right) \left(\frac{1 - h_{\mathrm{m}t+1} - h_{\mathrm{h}t+1}}{1 - h_{\mathrm{m}t} - h_{\mathrm{h}t}}\right)^{\omega(\sigma-1)} \\
&\qquad = \frac{\beta}{g_{\mathrm{m}t+1}^{\sigma}} \left[(1-\tau_{\mathrm{k}t+1})\,\alpha\tilde{k}_{\mathrm{m}t+1}^{\alpha-1} h_{\mathrm{m}t+1}^{1-\alpha} + 1 - \delta_{\mathrm{m}t+1}\right], && \text{(A. 24a)}\\
&\left(\frac{\tilde{\mathcal{C}}_{t+1}}{\tilde{\mathcal{C}}_{t}}\right)^{\sigma-1} \left(\frac{\tilde{c}_{\mathrm{m}t+1}}{\tilde{c}_{\mathrm{m}t}}\right) \left(\frac{1 - h_{\mathrm{m}t+1} - h_{\mathrm{h}t+1}}{1 - h_{\mathrm{m}t} - h_{\mathrm{h}t}}\right)^{\omega(\sigma-1)} \\
&\qquad = \frac{\beta}{g_{\mathrm{m}t+1}^{\sigma}} \left[\left(\frac{1-\nu}{\nu}\right)\left(\frac{\tilde{c}_{\mathrm{m}t+1}}{\tilde{c}_{\mathrm{h}t+1}}\right) \alpha_{\mathrm{h}}\tilde{k}_{\mathrm{h}t+1}^{\alpha_{\mathrm{h}}-1} h_{\mathrm{h}t+1}^{1-\alpha_{\mathrm{h}}} + 1 - \delta_{\mathrm{h}t+1}\right]. && \text{(A. 24b)}
\end{aligned}$$

In the above expressions, the home and market sectors have identical growth rates of labor-augmenting technology. As discussed by Gomme and Rupert (2007, pp. 463–464), there are several direct and indirect methods to impute the growth rate of labor-augmenting technology in the home sector, but each relies on unverifiable assumptions. Identical productivity growth rates also facilitates the analysis technically: when the long-run growth rates of productivity in the home and market sectors are different, and unless the elasticity of inter-temporal substitution is unitary (the logarithmic case), the model exhibits a *non*-balanced growth path; see Ngai and Pissarides (2007). Clearly, future work should quantitatively examine whether departures from identical long-run productivity growth rates in home and market sectors have economically significant impact on the saving rate.

Finally, there are two market-clearing equations:

$$\begin{aligned}
&\tilde{c}_{\mathrm{h}t} = \tilde{k}_{\mathrm{h}t}^{\alpha_{\mathrm{h}}}\, h_{\mathrm{h}t}^{1-\alpha_{\mathrm{h}}}, && \text{(A. 25a)}\\
&n_{t+1}\, g_{\mathrm{m}t+1}\left(\tilde{k}_{\mathrm{m}t+1} + \tilde{k}_{\mathrm{h}t+1}\right) \\
&\qquad = \tilde{k}_{\mathrm{m}t}^{\alpha} h_{\mathrm{m}t}^{1-\alpha} - \tilde{g}_t + (1-\delta_{\mathrm{m}t})\tilde{k}_{\mathrm{m}t} + (1-\delta_{\mathrm{h}t})\tilde{k}_{\mathrm{h}t} - \tilde{c}_{\mathrm{m}t}. && \text{(A. 25b)}
\end{aligned}$$

This completes the description of the optimal allocations.

The steady-state values of $h_{\mathrm{m}}^*, h_{\mathrm{h}}^*, \tilde{y}_{\mathrm{m}}^*, \tilde{y}_{\mathrm{h}}^*, \tilde{k}_{\mathrm{m}}, \tilde{k}_{\mathrm{h}}, \tilde{c}_{\mathrm{h}}$ and $\tilde{c}_{\mathrm{m}}$ satisfy

$$1 = \frac{\beta}{g_{\mathrm{m}}^{\sigma}} \left[(1-\delta_{\mathrm{m}}) + (1-\tau_{\mathrm{k}})\alpha \left(\frac{\tilde{y}_{\mathrm{m}}^*}{\tilde{k}_{\mathrm{m}}^*} \right) \right], \tag{A. 26a}$$

$$1 = \frac{\beta}{g_{\mathrm{m}}^{\sigma}} \left[(1-\delta_{\mathrm{h}}) + \left(\frac{1-\nu}{\nu} \right) \alpha_{\mathrm{h}} \left(\frac{\tilde{y}_{\mathrm{h}}^*}{\tilde{k}_{\mathrm{h}}^*} \right) \left(\frac{\tilde{c}_{\mathrm{m}}^*}{\tilde{c}_{\mathrm{h}}^*} \right) \right], \tag{A. 26b}$$

$$\frac{\tilde{y}_{\mathrm{m}}^*}{\tilde{k}_{\mathrm{m}}^*} = \frac{\tilde{c}_{\mathrm{m}}^*}{\tilde{k}_{\mathrm{m}}^*} + \tilde{g} - (1-\delta_{\mathrm{m}} - n\, g_{\mathrm{m}}) \left[1 + \frac{\tilde{k}_{\mathrm{h}}^*}{\tilde{k}_{\mathrm{m}}^*} \right], \tag{A. 26c}$$

$$\frac{h_{\mathrm{m}}^*}{1 - h_{\mathrm{m}}^* - h_{\mathrm{h}}^*} = \left(\frac{1-\alpha}{\omega} \right) \left(\frac{\tilde{y}_{\mathrm{m}}^*}{\tilde{c}_{\mathrm{m}}^*} \right), \tag{A. 26d}$$

$$\frac{h_{\mathrm{h}}^*}{1 - h_{\mathrm{m}}^* - h_{\mathrm{h}}^*} = (1-\alpha_{\mathrm{h}})\,(1-\nu), \tag{A. 26e}$$

$$\frac{\tilde{y}_{\mathrm{m}}^*}{\tilde{k}_{\mathrm{m}}^*} = (h_{\mathrm{m}}^*)^{1-\alpha} \left(\tilde{k}_{\mathrm{m}}^* \right)^{\alpha-1}, \tag{A. 26f}$$

$$\tilde{y}_{\mathrm{h}}^* = \left(\tilde{k}_{\mathrm{h}}^* \right)^{\alpha_{\mathrm{h}}} \left(h_{\mathrm{h}}^* \right)^{1-\alpha_{\mathrm{h}}}, \tag{A. 26g}$$

$$\tilde{c}_{\mathrm{h}}^* = \tilde{y}_{\mathrm{h}}^*. \tag{A. 26h}$$

Following the standard calibration methodology, using data I set the steady-state values of $h_{\mathrm{m}}^*, h_{\mathrm{h}}^*$, $\tilde{y}_{\mathrm{m}}^*/\tilde{k}_{\mathrm{m}}$, and $\tilde{k}_{\mathrm{h}}/\tilde{k}_{\mathrm{m}}$ as "targets," and work with the implied values of β, ω, ν, and α_{h} (see Table A.1), thereby maintaining eight equations in eight unknowns.

Table A.1: Steady-state values of exogenous variables, calibrated parameters, and calibration targets

Mnemonic	Value	Description and source
β	0.9409	Subjective discount rate; calibrated
σ	2	Inverse of the elasticity of inter-temporal substitution
ω	3.3052	Weight on leisure in the instantaneous utility function; calibrated
α	0.283	Share of capital in private market production; Gomme and Rupert (2007, Table 4 based on NIPA)
α_{h}	0.2881	Share of housing stock in home production; calibrated
ν	0.5445	Weight on home goods consumption in the consumer aggregator; calibrated
g_m	1.0081	Gross rate of labor augmenting technological change (steady state); Jorgenson et al. (2008, Table 2, baseline scenario) and α
n	1.0068	U.S. Census Bureau (2004, year 2050)
g	0.2059	Share of government spending in output; NIPA
τ_{k}	0.2284	Tax rate on capital income (ad valorem); NIPA
τ_{ℓ}	0.2121	Tax rate on labor income (ad valorem); NIPA
δ_{m}	0.0673	Depreciation rate of market capital; NIPA
δ_{h}	0.0121	Depreciation rate of housing; NIPA
h^*_{m}	0.2364	Share of time spent in market production; BLS
h^*_{h}	0.096	Share of time spent in nonmarket production; Aguiar and Hurst (2007, Table 2)
$\tilde{k}_{\mathrm{m}}/\tilde{y}_{\mathrm{m}}$	1.4144	Market capital to output ratio; NIPA and BEA
$\tilde{k}_{\mathrm{h}}/\tilde{k}_{\mathrm{m}}$	0.9579	Housing stock to market capital ratio; NIPA

Notes: See the data appendix for the description of variables calculated from the Bureau of Economic Analysis (BEA), National Income and Product Accounts (NIPA), and Bureau of Labor Statistics (BLS).

B Data sources for parameters and variables

Income, and consumption

The measure of income in Figure 1 is nominal output from (NIPA Table 1.1.5), and consumption is nominal personal consumption expenditures (NIPA Table 2.3.5). In section 4, the income concept is nominal market output,

$$Y_{\mathrm{m}} = \text{nominal output} - \text{gross housing value added (NIPA Table 1.3.5)},$$

and the consumption concept is nominal market consumption,

$$C_{\mathrm{m}} = \text{personal consumption expenditures} - \text{gross housing value added.}$$

For government expenditures, G, I use government consumption expenditures and gross investment (NIPA).

Capital stock

Current-cost net stock of private fixed assets and consumer durable goods are from the Bureau of Economic Analysis (BEA, Tables 1.1 and 2.1). Market capital, K_{m}, is the sum of nonresidential structures, equipment and software. Since the BEA reports fixed assets on a year-end basis, in the quantitative analysis the value of capital stock for year t in the corresponding value in BEA tables for year $t-1$.

Tax rate on capital

I calculate the tax rate on *corporate* profits and use it as a measure of the *ad valorem* tax rate on fixed capital income:

$$\tau_{\mathrm{k}} = \frac{\text{taxes on corporate income}}{\text{corporate profits before taxes} + \text{consumption of fixed capital}} \tag{B. 1}$$

In calculating τ_{k}, consumption of fixed capital (depreciation) is added to corporate profits to determine corporate income. Mendoza et al. (1994) and Poterba (1998) compute corporate capital income tax rates in a similar way. Their denominator, however, is simply corporate profits before taxes with IVA and CCAdj. These differences should be kept in mind when comparing alternative estimates of tax rates and after-tax rates of return on capital.

Corporate profits before taxes. As in the NIPA, corporate profits before corporate taxes from *current* production is measured as the sum of profits before taxes, inventory valuation adjustment (IVA), and the capital consumption adjustment (CCAdj). Both the IVA and CCAdj removes from profits the capital-gain-like element or the capital-loss-like element that results from inventory withdrawals and depreciation of fixed assets at the prices of earlier periods. The BEA deducts property taxes paid by corporations from corporate earnings before reporting corporate profits before taxes in the NIPA. Source: NIPA Table 1.13.

Consumption of fixed capital. In the NIPA the CCAdj is the difference between consumption of fixed capital (CFC) and capital consumption allowances. CFC is a charge for the depreciation of fixed capital. The BEA calculates this based on studies of prices of used equipment and structures in resale markets. Capital consumption allowances consist of tax-return-based depreciation charges for corporations. Thus, the appropriate current-cost based depreciation is CFC calculated as the sum of corporate CCAdj and corporate sector capital consumption allowance; see also U.S. Department of Commerce (2002). Sources: NIPA Tables 1.13, and 6.22.

Taxes on corporate income. This consists of taxes on corporate income paid to federal, state, and local governments, but excludes property taxes levied on corporations. Source: NIPA Table 6.18.

Discussion. There is no straightforward method to calculate the tax rate on capital, so any imputation of this tax rate involves judgement. The method I outlined above is simple, and takes a narrow view of capital, both in terms of sector coverage and type of assets: It only covers the corporate sector and excludes land and other tangible assets. It focuses on the corporate sector because this introduces fewer assumptions into the analysis. It excludes land for the purposes of consistency with the BEA's capital stock and depreciation estimates.

There are, of course, alternative methods. Poterba (1998) sums corporate income taxes, property taxes and investor taxes (taxes on dividends and interest income) to arrive at a comprehensive measure of tax burden on corporate earnings (non-financial corporations only). Gomme and Rupert (2007) calculate the tax rate on capital, τ_k by first *imputing* "taxes paid"

$$\begin{aligned}\text{taxes paid } &= \tau_h\,(\text{net interest} + \alpha\,\text{proprioter's income} + \text{rental income}) \\ &\quad + \text{taxes on corportate capital} \\ &\quad + \text{state and local property taxes} \\ &\quad + \text{state and local other taxes,}\end{aligned} \tag{B. 2}$$

where the personal tax rate, τ_h is

$$\tau_{\mathrm{h}} = \frac{\text{personal current taxes}}{\text{wages and salaries} + \text{proprietors' income} + \text{rental income} + \text{asset income}}, \tag{B. 3}$$

where 'asset income' includes dividend and interest income. To obtain τ_k divide these taxes paid by the total income from the corresponding sources

$$\tau_{\mathrm{k}}^{\mathrm{gr}} = \frac{\text{taxes paid}}{\text{net operating surplus} + \text{capital consumption} - (1-\alpha)\,\text{proprietor's income}}, \tag{B. 4}$$

where α is the share of capital in income.

These methods make an allowance for taxes on capital that originate from interest income. However, the classification of property taxes entirely as a tax on capital services assumes that these taxes do not pay for any service that is economically valuable for the corporation. At the same time, these methods allocate a variety of taxes to capital services by *imputing* taxes on rental income, interest income, and share of proprietor's income.

Rate of return on capital

To maintain consistency with the tax rate on capital, I compute the before tax rate of return on *corporate* capital, $r_{\mathrm{k,C}}$, as

$$r_{\mathrm{k,C}} = \frac{\text{corporate profits before taxes + net interest}}{\text{corporate fixed assets}}, \tag{B. 5}$$

where corporate profits are with IVA and CCAdj.

I compute the model-based before-tax rates of return, r, as

$$r = \alpha(\tilde{y}_{\mathrm{m}}/\tilde{k}_{\mathrm{m}}) - \delta_{\mathrm{m}}, \tag{B. 6}$$

and compare them with estimated $r_{\mathrm{K,C}}$.

Corporate fixed assets. These are current-cost, net stock of private fixed assets reported by BEA in NIPA Table 6.1. Since the BEA reports fixed assets on a year-end basis, for corporate capital stock in year t, I used the corresponding value in the BEA tables for year $t-1$.

Discussion. In the literature, it is common to compute after-tax rates of return on capital and compare them with existing estimates (e.g, those calculated by Poterba (1998)). In our theoretical model,

$$\text{model-based after-tax rate of return} = (1-\tau_{\mathrm{k}})\alpha(\tilde{y}_{\mathrm{m}}/\tilde{k}_{\mathrm{m}}) - \delta_{\mathrm{m}}. \tag{B. 7}$$

However, these are not immediately comparable with the existing estimates for two reasons. First, Poterba (1998, Table 1) uses an income measure for calculating the tax rate on capital that excludes consumption of fixed capital. A comparable after-tax rate of return would, therefore, require constructing

$$(1-\tau_{\mathrm{k}}^{\mathrm{net}})\left[\alpha(\tilde{y}_{\mathrm{m}}/\tilde{k}_{\mathrm{m}}) - \delta_{\mathrm{m}}\right],$$

where tax rate on corporate income is calculated as

$$\tau_{\mathrm{k}}^{\mathrm{net}} = \frac{\text{taxes on corporate income}}{\text{corporate profits before taxes}}. \tag{B. 8}$$

Second, Poterba's rate of return series are for nonfarm nonfinancial corporate business sector *and* he calculates the return on all tangible assets, including structures, equipment software, land and inventories. The BEA estimates fixed assets (structures, equipment and software) at current (replacement) cost, whereas nonfarm nonfinancial corporate business sector tangible assets are measured at market value, historical cost, and replacement cost. To calculate the rate of return, I use (i) BEA's fixed assets because the model does not have land or inventories, and (ii) corporate fixed assets because the capital stock in the model does not distinguish between financial and nonfinancial firms. In fact, at the end of the sample, in the nonfarm nonfinancial corporate business sector the ratio fixed assets (from BEA) to tangible assets (from FFA at market cost) has been significantly below 0.8, which largely accounts for the rate of return differentials reported in this paper and those reported in Poterba (1998, Table 1 column 1).

Tax rate on labor income

After-tax labor income. This is labor income minus labor taxes. I compute labor income, Y^{L}, as (all line references below are to NIPA Table 2.1):

$$\begin{aligned}\text{labor income} = {} & \text{wages and salaries (line 3)} \\ & + \text{personal current transfer receipts (line 16)} \\ & - \text{personal contributions for government social insurance (line 24)},\end{aligned} \tag{B. 9}$$

and labor taxes, τ^{l}, as:

$$\text{labor taxes} = \text{personal current taxes (line 25)} \times \text{share of labor taxes} \tag{B. 10}$$

where

$$\text{share of labor taxes} = \frac{\text{wages and salaries}}{\text{wages and salaries} + \text{nonwage income}}, \tag{B. 11}$$

where

$$\begin{aligned}\text{nonwage income} = {} & \text{proprietors' income (line 9)} + \text{rental income (line 12)} \\ & + \text{interest income (line 14)} + \text{dividend income (line 15)}.\end{aligned} \tag{B. 12}$$

Individual and payroll tax on labor income. I use individual and payroll tax rate on labor income as a measure of tax rate on labor income, τ_ℓ:

$$\tau_\ell = \frac{\tau_{\mathrm{h}} \times \text{wages and salaries} + \text{contributions for social insurance}}{\text{wages and salaries} + \text{employer contributions for social insurance}}, \tag{B. 13}$$

where the personal tax rate, τ_{h} in equation (B. 3).

Discount factor

In several cases, I calibrate the subjective discount factor using an estimate of the real interest rate. Specifically, in examples 1 and 2, I use an annualized discount rate of 6.5%, which corresponds to $\beta = 0.939$. (I use a common discount rate for illustrative purposes: as identical balanced growth restrictions imply different discount rates in models with and without capital income taxes.)

A common alternative is to use the restrictions implied by the balanced growth path. Specifically, equation (A. 15a)—or (A. 26a)—implies that

$$g_{\mathrm{m}}^{\sigma} = \beta \left[(1-\delta_{\mathrm{m}}) + (1-\tau_{\mathrm{k}})\alpha \left(\frac{\tilde{y}_{\mathrm{m}}^*}{\tilde{k}_{\mathrm{m}}^*} \right) \right], \tag{B. 14}$$

and it is common practice to calibrate the subjective discount rate by using parameter values on $g_{\mathrm{m}}, \delta_{\mathrm{m}}, \tau_{\mathrm{k}}, \alpha$, as well as the so-called "calibration target" output-capital ratio, $\tilde{y}_{\mathrm{m}}^*/\tilde{k}_{\mathrm{m}}^*$. In general, it is not straightforward to choose the output-capital ratio. For instance, Chen et al. (2007) simply set it equal to two for Japan, and do not discuss this choice further. Gomme and Rupert (2007) link it to sample average of the investment-output ratio, which implicitly assumes that the throughout the sample period all variables of interest exhibit only transitory deviations from their balanced growth paths.

Elasticity of inter-temporal substitution

In the model $1/\sigma$ corresponds to the elasticity of intertemporal substitution. In the simulations, I set $\sigma = 2$ (a "low" elasticity) and also discuss the sensitivity of the results to $\sigma = 1$.

Share of capital

The share of capital is the elasticity of market output with respect to capital, and is the sum of share of structures, equipment and software in market value added: $\alpha = 0.283$. Source: Gomme and Rupert (2007, Table 4).

Depreciation rate

I calculated the implied depreciation rate on private *nonresidential* fixed assets (equipment, software and structures), δ_{m}, as follows: Current-cost depreciation divided by end-of-period current-cost net stock of assets. Source: Bureau of Economic Analysis (Private Fixed Assets by Type, Tables 2.1 and 2.4).

Growth rate of technology

1952–2006 (revised data). To determine the growth rate of labor augmenting productivity factor in the market sector, g_m, I use the estimates of multi-factor productivity (MFP) in the private business sector, excluding government enterprises (Bureau of Labor Statistics (BLS) series ID: MPU740023 (K)), and calculate the gross growth rate as

$$g_{\mathrm{m}t} = (\mathrm{MFP}_t/\mathrm{MFP}_{t-1})^{1/(1-\alpha)}, \tag{B. 15}$$

where α is the elasticity of capital with respect to output. These estimates are based on the recent revised data, and as such, might have different from productivity growth forecasts using real-time data.

1965–2006 (real-time data). Edge et al. (2007, Table 1) compute estimates of long-run labor productivity growth rate in the nonfarm business sector using Kalman filter and real-time data from 1965 to 2005. To compute the real-time labor augmenting productivity growth rate, I first subtract the contributions of capital deepening and labor quality from labor productivity to convert these into estimates total factor productivity growth rate. The forecasts of capital deepening and labor quality are not available, and I use the estimates—based on revised data—reported in Jorgenson, Ho, and Stiroh (2008, Table 1).[28]

Since 1969 the Council of Economic Advisors (CEA) in the *Economic Report of the President* have annually been publishing their forecasts of long-run (6 year horizon) labor productivity growth in the nonfarm business sector. I also used the CEA's forecasts, and found that the results are indistinguishable from those of Kalman filter estimates of Edge et al. (2007). Since the CEA and Edge et al. (2007) estimates do not cover identical years, I (arbitrarily) impute

[28] The Congressional Budget Office's (CBO) *The Budget and Economic Outlook* annually publishes separate forecasts of labor and total factor productivity in the nonfarm business sector. However, these estimates are not available before 1996.

missing values to span the same time period. The CEA forecasts started in 1969 and did not report forecasts in 1972, 1973, 1975, 1976, 1981, and 1982, so I use the Edge et al. real-time Kalman filter estimates corresponding to those "missing" years. Edge et al. estimates end in 2005, so I use the CEA forecast for 2006.

2006–2016 (forecast). I use the private nonfarm business sector total factor productivity growth rate projections by Jorgenson, Ho, and Stiroh (2008, Table 2) and convert them into gross growth rate of labor augmenting productivity factor as above. Their "pessimistic" case is 0.50 percent per year, "base" case is 0.81 percent, and "optimistic" case is 0.95 percent.[29]

Population

1952–2006. I calculate the growth rate of population, $n = N_t/N_{t-1}$ based on the civilian noninstitutional population, ages 16 years and over (BLS, series ID: LNU00000000).

2007–2050 (forecast). Population projections for ages 16 years and over are from U.S. Census Bureau (2004) These projections are very similar to the United Nations's (2006) population projections based on constant fertility assumption.

Hours worked

Market hours. I constructed h_{m} as the ratio of hours worked to total discretionary hours:

$$h_{\mathrm{m}} = \frac{\text{weekly market hours worked per person}}{\text{total discretionary hours per week per person}}, \tag{B.16}$$

where

$$\begin{aligned}&\text{weekly market hours worked per person}\\ &\qquad = \text{average weekly hours} \times \text{market employment rate},\end{aligned} \tag{B.17}$$

and

$$\text{market employment rate} = \frac{\text{civilian employment}}{\text{civilian noninstitutional population}}. \tag{B.18}$$

Sources: U.S. Department of Labor, Bureau of Labor Statistics, and accessed through FRED (Federal Reserve Economic Data) Link: http://research.stlouisfed.org/fred2, Economic Research Division Federal Reserve Bank of St. Louis (series IDs in parentheses). *Average weekly hours* is for total private industries (AWHNONAG). These series start in 1964, so I constructed the series for 1952–1963 scaling the average weekly hours, manufacturing (AWHMAN) by the ratio of manufacturing hours to total private industries hours in 1964. (Over the balance of the sample series this ratio increases secularly.) These data are monthly and seasonally adjusted. *Civilian employment* (CE16OV) is monthly, seasonally adjusted, and includes persons 16 years

[29]The CBO's estimates of total factor productivity in the nonfarm business sector do not control for labor quality so I do not use them as they are not comparable with the BLS and Jorgenson et al. estimates.

of age and older. *Civilian noninstitutional population* (CNP16OV) is monthly, not seasonally adjusted and includes persons 16 years of age and older. Total discretionary hours per week is 90, defined as total hours within a week (168 hours) minus hours for eating, sleeping, personal activities, such as hygiene, and shopping (78 hours, discussed below). Annual values are the monthly averages of h_{m} given in equation (B. 16). This definition of market hours corresponds to the "core market work" in Aguiar and Hurst (2007). Sample average of h_{m} is 0.236, which is slightly less than the corresponding value of 0.255 reported by Gomme and Rupert (2007, Table 5) based on the *American Time-Use Survey*, 2003.

Home hours. There are significant conceptual difficulties associated with determining hours spent in home production (nonmarket work hours). I associate home production with cooking, cleaning, indoor painting, and other household chores done inside the house. I include child care and gardening in the residual category "leisure." Each of these classifications is subjective. An additional complication is that there are no estimates of nonmarket hours work based on nationally representative surveys prior to 1965. The *American Time-Use Survey*, available for 1965, 1975, 1985, 1993, and 2003, provide the most detailed window on nonmarket hours. In the absence of annual data spanning 1952–2006, I used 1965 values for years 1952–1964, and 2003 values for 2004 and beyond. Table B.1 shows the existing estimates, based on Aguiar and Hurst (2007), for average weekly hours spent in (core) nonmarket work. There is a secular decline in the hours spent in nonmarket work.[30]

[30] The average hours of work spent in market and nonmarket work reported in Table B.1 are for a nationally representative sample of individuals aged between 21 and 65. As such, Aguiar and Hurst's sample excludes individuals 16 through 20 years of age, and 66 and over, which are included in the population measure I use in this study. Using Aguiar and Hurst's average hours per week spent in nonmarket work as a fraction of total discretionary hours per week is thus problematic to the extent that the individuals excluded from their sample have a different time allocation for nonmarket work. The advantage, on the other hand, of their approach is that their data on hours worked control for demographic changes that have taken place over time. Since such demographic considerations are absent in the model here, their estimates are appropriate in our context. Aguiar and Hurst also report average time spent in market work, but since their sample excludes those who are more likely to be nonemployed, their numbers for h_{m} are not suitable for my purposes.

Table B.1: Nonmarket hours of work from *American Time-Use Surveys*

Time-use category	1965	1975	1985	1993	2003
Average hours per week spent in					
core nonmarket work	13.02	11.34	10.82	8.75	8.66
nondiscretionary hours	77.64	78.78	78.88	77.77	77.59
As a fraction of total discretionary hours per week					
Core nonmarket work	0.145	0.126	0.120	0.097	0.096

Source: Aguiar and Hurst (2007, Tables 2 and 9); full sample.
Notes: Total discretionary hours is 90, and defined as total hours within a week (168 hours) minus nondiscretionary hours, including eating, sleeping, personal activities and shopping (78 hours). Nondiscretionary hours is computed as Aguiar and Hurst's Leisure 1 measure minus their Leisure 2 measure plus time spent in obtaining goods and services/shopping. Core nonmarket work includes food preparation, food presentation, kitchen/food cleanup, washing/drying clothes, ironing, dusting, vacuuming, indoor cleaning, indoor painting, etc. Aguiar and Hurst's full sample covers individuals aged between 21 and 65, and averages for each year control for demographic changes from 1965 to 2003.

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